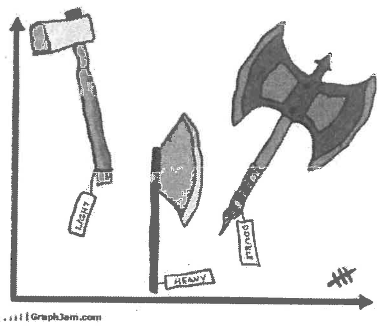
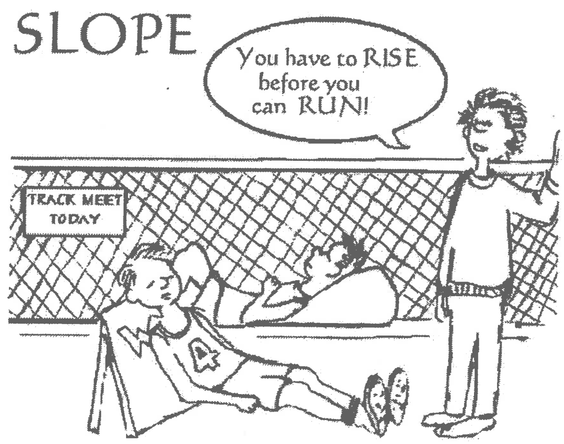
Unit 2:

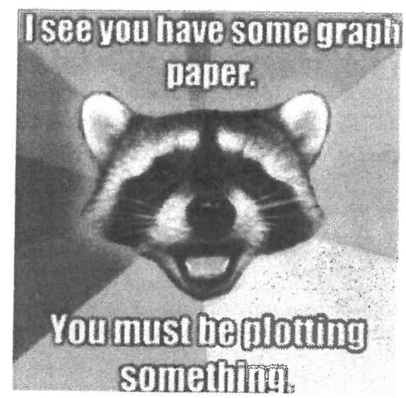
Relations and Patterns

Lessons and Exercises

Always label your axes







Name: notes + answers

lesson	assignment	dates
1.scatterplots (p. 2-14)	p. 12-14	
2. linear patterns (p. 15-18)	p. 17-20	
3.equations of linear relations (p. 19-29)	p. 27-29	
4.the slope of a line (p. 30-37)	p. 36-37	
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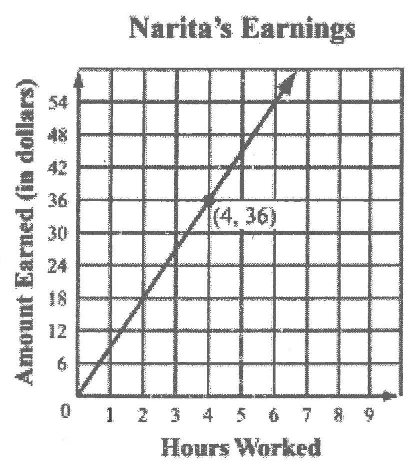
Test		

Introduction:

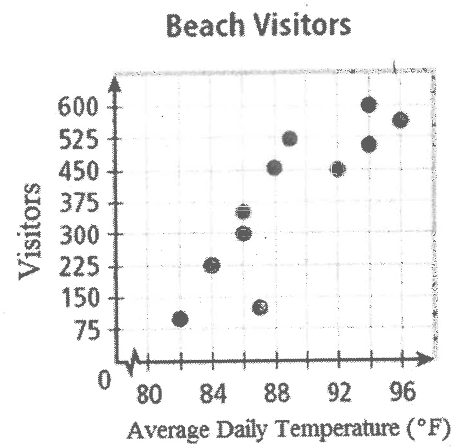
This unit focuses on mathematical relationships that are linear, such as the *number of hours you work* and the *amount of money you earn*. The amount of money earn **DEPENDS** on the number of hours you work. *A change in one quantity (hours worked)* **affects** the change in another quantity (money earned).

Mathematical relationships are expressed in 4 ways:

1. Word statements
2. Table of values
3. Equations
4. Graphs



Lesson #1: Scatterplots



In this lesson, you will:

- Identify discrete and continuous data
- Draw scatterplots from data
- Anticipate the shape of a graph that represents a relationship

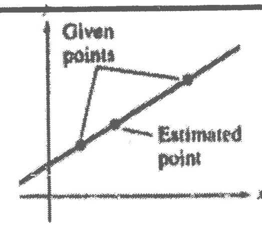
Scatterplots

Often, when real-world data is plotted, the result is a linear pattern. The general direction of the data can be seen, but the data points do not all fall on a line. This type of graph is called a scatter plot. A scatter plot **is a graph of plotted points that shows the possible relationship between two data sets**. (*in the example above, each dot represents the number of visitors for a given temperature*). A scatter plot is often used to investigate whether or not there is a relationship or connection between 2 sets of data. A scatterplot consists of an **x-axis** (the horizontal axis), a **y-axis** (the vertical axis), and a series of dots. Each dot on the scatterplot represents one observation from a data set (it shows the relationship between two sets of data). The position of the dot on the scatterplot represents its X and Y values. You can look at the overall pattern of the points on a scatter plot to see if there is a relationship between the 2 sets of data, it will be easy to see if the data is plotted on a scatter plot – to see if they show a trend. A scatter plot is an effective way to show some types of data.

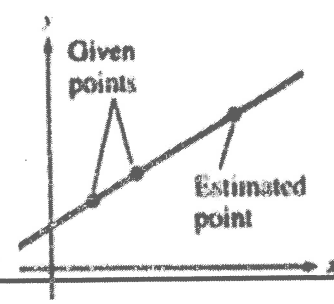
Displaying data in a graph can help you to see relationships between two sets of data. **A graph is a visual representation of a numerical relationship**. It is possible to **predict** the value of one variable when the other value is known.

This prediction is called **interpolation** or **extrapolation,** depending whether the prediction is **between** two known values or **beyond** the known values.

- **Interpolation** is estimating values **BETWEEN** **the existing data points** in the graph.



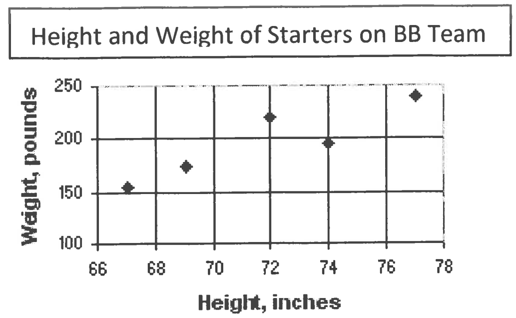
- **Extrapolation** is estimating values **OUTSIDE** **the existing data points** in the graph. Follow the pattern of the values to extend the graph to predict a value not on the graph.



To see a relationship and to make predictions we first must make a **graph** that relates the two variables. **Here is an example of a graph of a scatter plot.**

Height, inches	Weight, pounds
67	155
72	220
77	240
74	195
69	175

←We have a **table of values** on the left with the height and the weight of five starters on a high school basketball team.



To the right, the same data are displayed in a **scatterplot.**↑

Axes: Independent and Dependent Variables

Creating a scatterplot is very similar to drawing a line graph.

The variable on the ***horizontal*** or ***x-axis*** is the ***independent variable*** because it is ***not*** affected by the other variable. The variable on the ***vertical*** or ***y-axis*** is the ***dependent variable***, because it DEPENDS on the independent variable. If we were looking at temperature at certain times of day, time would be the independent variable because time does not depend on temperature. The temperature (*dependent variable*) DEPENDS on the time of day (*independent variable*).

Example: Identify the dependent (d) and independent (i) variables in each statement:

a) The **size of a dog** compared to **the amount of food he eats**

b) The **amount you earn** compared to **the number of hours you work**

c) The **amount of fertilizer used** compared to the **plant growth**

Creating a Good Scatterplot:

A good scatterplot requires the following: for a relationship between two things you are trying to find out what makes the dependent variable change the way it does. **Look at the graphs above to see all these parts represented.**

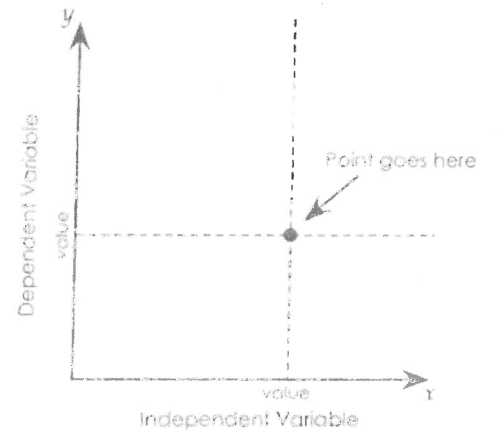
- A title
- Labels for the **axes**
- Scales for the axes
- **Accurately** plotted points representing the ordered pair

On your resource sheet, list the **steps to a good scatterplot.**

<u>Steps to Drawing a Scatterplot</u> *(see example below)*

1. Draw and $\boxed{\textbf{label}}$ the horizontal and vertical axes correctly (see p. 6) with the $\boxed{\textbf{variables}}$ and $\boxed{\textbf{units}}$ they were measured in (if there are units).

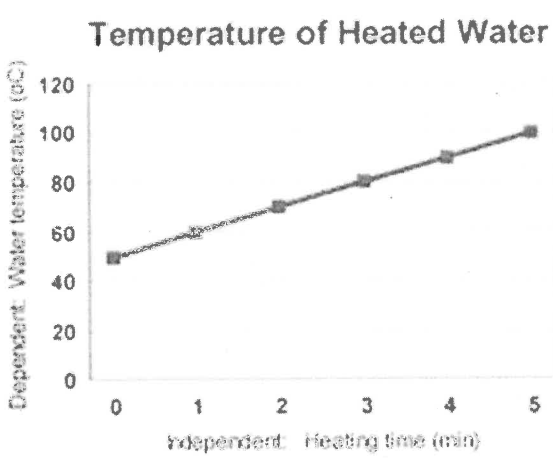
2. Write a suitable $\boxed{\textbf{scale}}$ on each axis. (The scale should allow you to <u>**fit all your given data**</u> on the graph and should allow the graph to <u>**fill most of the space**</u> given. Choose divisions on the axes that make it <u>easy to plot the points accurately</u>. The divisions and the scale should be <u>**consistent**</u> for the whole axis.



3. **Plot each point** accurately and clearly by finding the location of the *independent* variable on the *x-axis,* and then the location of the *dependent* variable on the *y-axis.*

4. Give the graph a $\boxed{\textbf{title}}$ that explains the relationship between the two variables.

Example

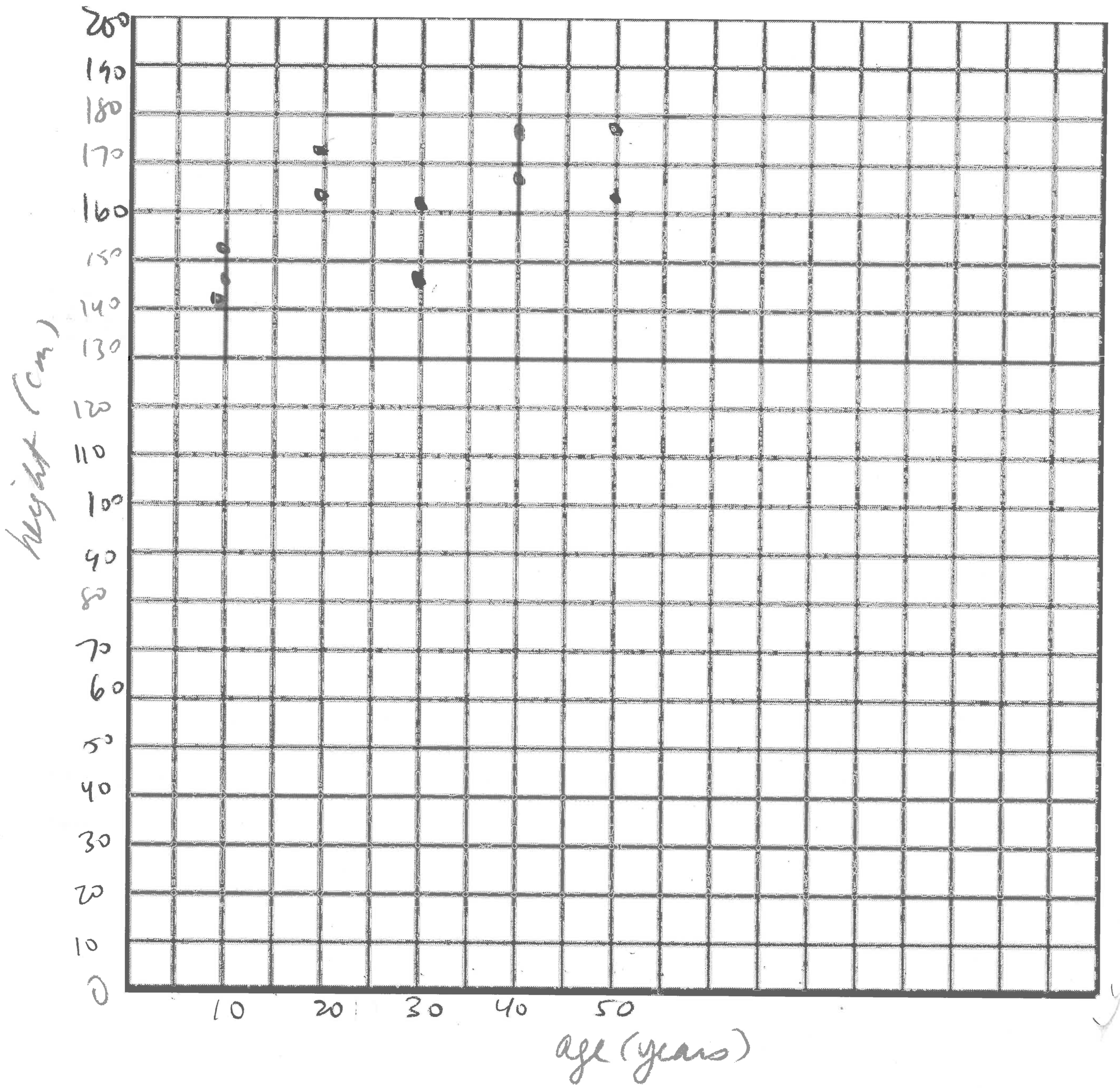


- *What is the title?* Temp of Heated Water
- *What does the x-axis represent? The y-axis?* heating time (min) Water temp (°C)
- *What is the independent variable?* heating time
- *What is the dependent variable?* water temp
- *What can we conclude from looking at this graph?* As the time increases, the temp. of water increases.

1. **Try it:** Given the following data set, draw a scatterplot. Include all the steps (see p.6).

Age in years	50	10	20	30	20	40	50	10	30	40	10
Height in cm	162	152	173	147	163	168	178	142	162	178	147

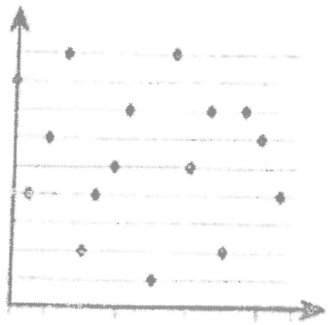
ages + heights



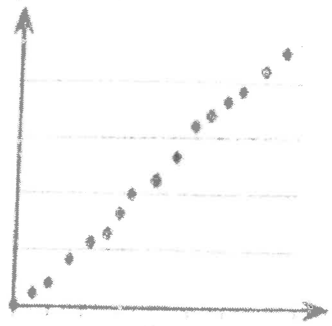
Checklist:
- Graph title ☐
- consistent scales on both axes ☐
- Titles on both axes ☐
- consistent division on both axes ☐
- Units on both axes ☐
- Scale chosen so it's easy to plot points ☐
- Scales chosen well so that space given is mostly filled with graph ☐

Linear and Non-Linear Patterns

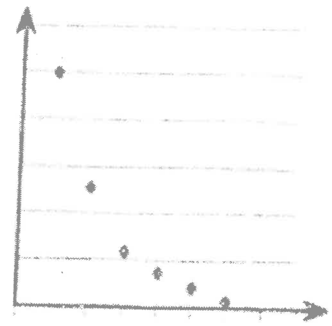
One pattern of special interest is a **linear pattern**, where the data has a general look of a line going uphill or downhill. Data can be linear or non-linear or have no pattern at all.



A



B



C

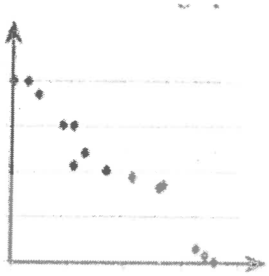
Graph A has no pattern. The data shows **no relationship, correlation, or trend.**

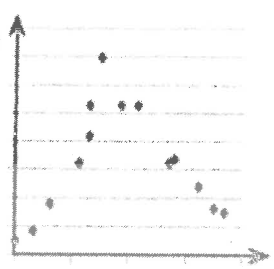
Graph B is **linear**. Although the points are not exactly in a straight line, you can see that it is close to being a straight line.

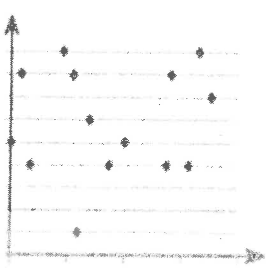
Graph C is non-linear because it is not a straight line. It does however show a distinct pattern and you could still predict (with some accuracy) where the next point should go.

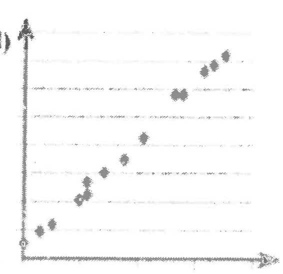
On your resource sheet, put an example of positive trend, negative trend, no trend, linear pattern, non-linear pattern.

2. Try it. – State whether each graph is linear, non-linear, or does not show any pattern. As well state if it shows a positive trend, a negative trend, both positive and negative trend, or no trend.

a) 

b) 

c) 

d) 

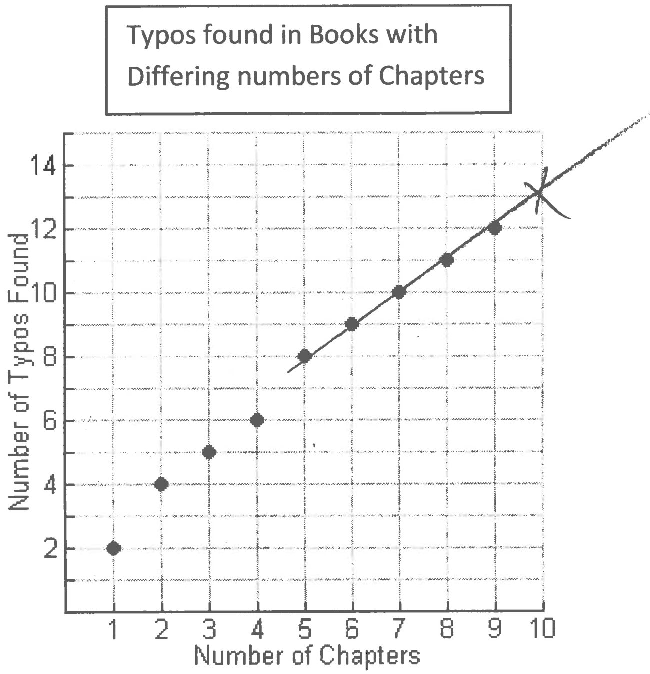
a) linear (sort of forms a line)
negative trend (decreasing)

b) non-linear
positive, negative trend

c) no pattern - seems random

d) linear - points form a line
positive trend

3. Try it.



a) What was the approximate number of typos found in a book with 4 chapters?

6

b) Approximately how many more typos were found in a book with 8 chapters than with 5 chapters?

8 Chapters → 11 typos ; 5 chapters → 8 typos || 11-8 = 3 more

C) Describe in words the trend represented by the scatter plot.

As the number of chapters increased, the number of typos found increased.

This is a positive trend/relationship/correlation.

The points have a linear pattern.

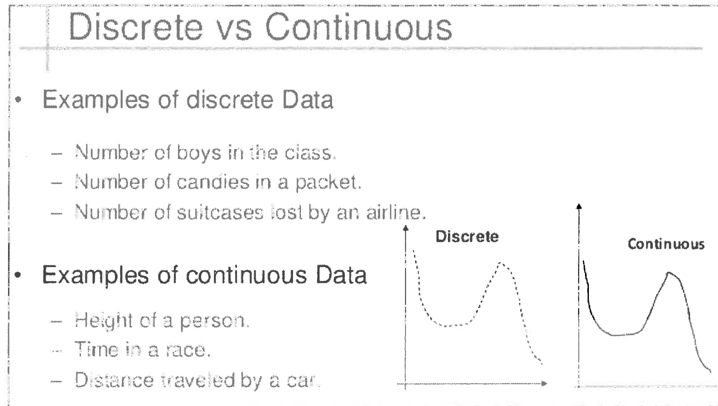
d) Predict how many typos might be found in a book with 10 chapters. 13

(Is this prediction called **interpolation** or **extrapolation**?)

e) Could this trend continue indefinitely? no- It will end when the book ends (finite # of chapters)

Continuous or Discrete Data in a scatter plot - line or dots?

Once we plot the data points we decide if the data is continuous or discrete. If it's **discrete** (there is no data possible between the points) then we **leave the graph as points**. If the data is **continuous** (data exists between the given points) then we **join the points** with a line or a curve.



↑Separate Points

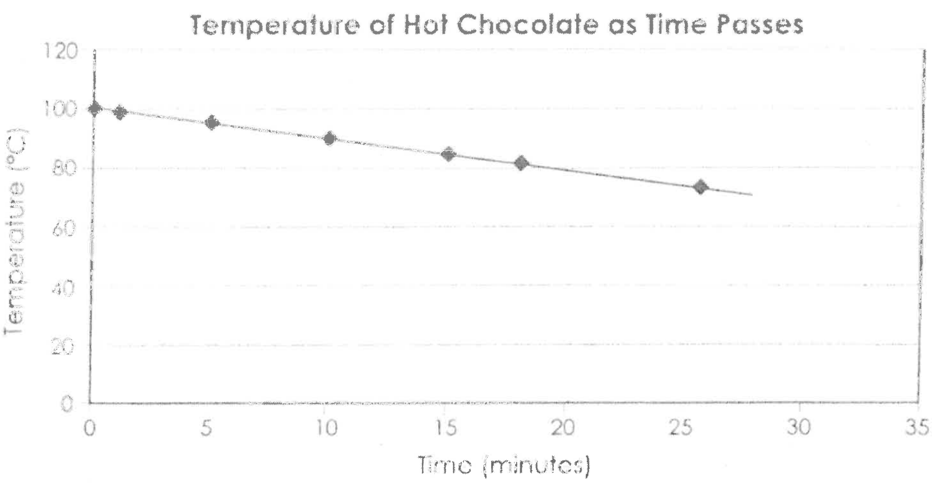
↑Points Joined

Example:

Goldie Locks hates it when she burns her mouth on hot chocolate in winter, but she also doesn't like to drink it cold. Goldie decides to conduct an experiment to find out how long she should wait before she drinks her hot chocolate, so that it is not too hot and not too cold. Draw a graph of the data.

Time Elapsed (minutes)	0	1	5	10	15	17	26
Temperature of the Hot Chocolate (°C)	100	99	95	80	85	83	74

Solution



The points are joined in this graph because time and temperature exists in between the times and temperatures given. The temperature and time data is continuous.

We join the dots with a line of best fit. In this case the points are all on the line. This will not always be the case. It is okay if some of the points are slightly beside the line.

Anticipating Graphs – Predict the Shape of a Graph based on the Data Given

To **predict the shape of a graph** that best describes the data, you need to answer the following questions:

- Is the graph going to **increase** or **decrease** from left to right?
- Is the data **discrete** or **continuous**?
- Is the data going to be in the shape of a straight **line** or a **curved line**?

Write these questions about **shape of graph prediction in your resource sheet.**

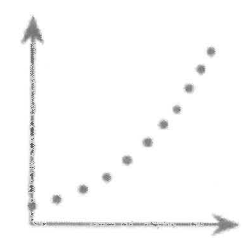
5. Try it: Predict the shape of the graph that best describes the data. Choose whether the data is continuous or discrete and say why. For each, write in the box how you know that's the shape.

A) The height of a ball at the top of each bounce after it has been dropped.
continuous / discrete? – why? only 1 max height per bounce

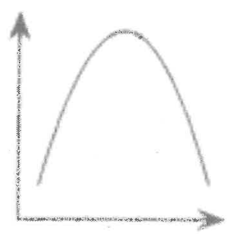
B) Your speed when you drop into a half pipe and come up on the other side.
continuous / discrete? – why? speed changes continuously

C) The amount of money you earn compared to the number of hours you work.
continuous / discrete? – why? you can be paid partial dollars for partial hours

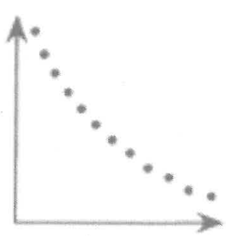
D) The value of an investment, with interest compounded annually, compared with the number of years the money is invested.
continuous / discrete? – why? Specific amounts of interest at certain times of year

D 

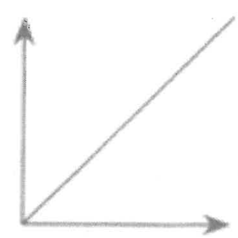
Amount of interest increases with time interest is compounded

B 

Speed increases rapidly as you go down then decreases rapidly as you go up

A 

height of ball decreases with each bounce once it has been dropped.

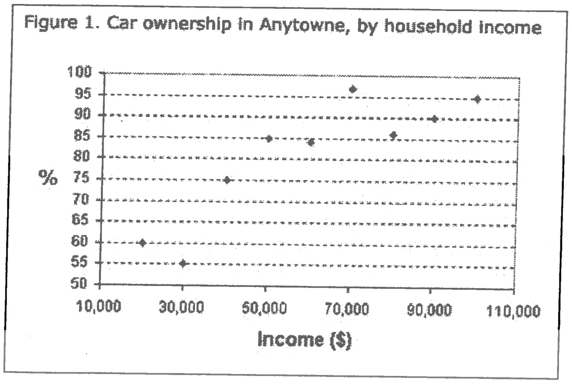
C 

as you spend more time working you make more money

Scatterplots Exercises

1. For each of the following scatterplots, check off the applicable descriptions for the trend shown.

a)



Linear ____

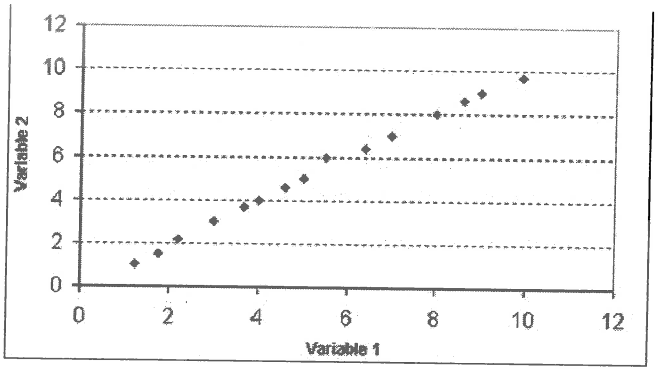
Non – linear ✓

Positive trend ✓

Negative trend ____

No trend ____

b)



Linear ✓

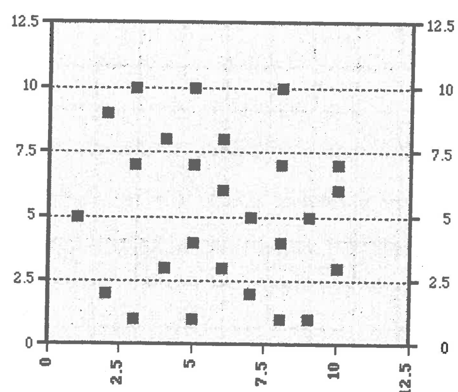
Non – linear ____

Positive trend ✓

Negative trend ____

No trend ____

c)



Linear ____

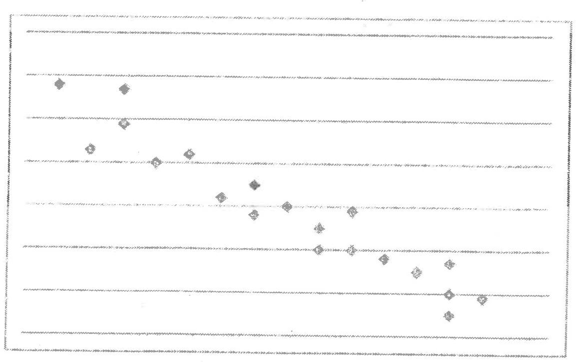
Non – linear ____

Positive trend ____

Negative trend ____

No trend ✓

d)



Linear ✓

Non – linear ____

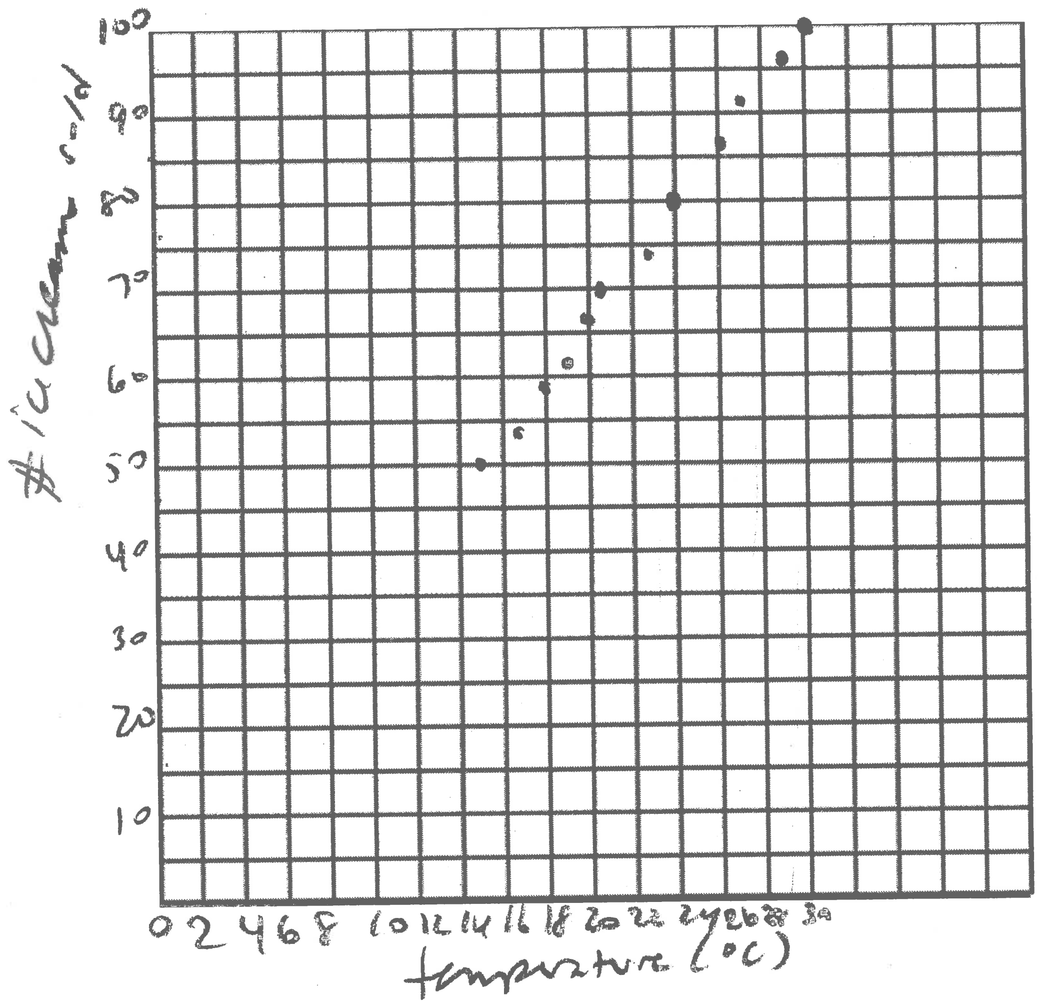
Positive trend ____

Negative trend ✓

No trend ____

2. Matt sells ice-creams at outdoor events. He often buys too much or too little ice-cream from the wholesalers, so does not make as much profit as he would like. He decides to record how many ice-creams he sells over a number of days, to see whether there is a link between the temperature and number of ice-creams sold. Here are his results. **Construct a scatter plot** to see if there is a relationship between the temperature and number of ice creams sold. Follow all the steps in creating a scatter plot (see p. 2). **Should you connect the dots of your points? Why or why not?** no - this is the total per day - there are no numbers in between temperatures.

Temperature (°C)	21	26	15	24	18	29	20	27	23	17	30	19
Number of ice-creams sold	70	86	50	80	58	96	66	92	74	54	100	62



Circle/fill in the blanks:

a) Would you say there is a **positive/negative , linear/non-linear** relationship, or **no relationship** between the temperature and number of ice creams sold?

b) It appears there **is / is not** a relationship between the temperature and number of ice creams sold. When the temperature is **low**, the number of ice creams sold is low. When the temperature is high, the number of ice creams sold is **high**. There is a positive linear trend.

3. For each relationship, state the **independent** and **dependent** variables *(see p. 3)*

a) The cost of a brand of cell phone compared to its memory.

independent: memory dependent: cost

b) The processing speed of your computer compared to the memory available on the hard drive.

independent: memory available dependent: processing speed

4. Which of the following situations are **continuous**? (see p. ~~9~~ 11) Why?

a) the final exam mark and average quiz marks for the students in a grade 11 math class
discrete - each mark for individual person

b) the price of a vehicle compared to its age continuous - vehicle can be valued at any price. car ages continously.

c) productivity of a factory and the number of workers working
discrete - you can only have whole # (no decimals) of worker

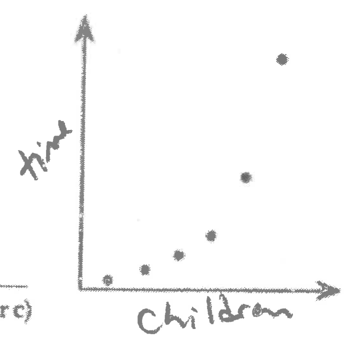
challenge 5. Match each situation with the most appropriate graph. Place **(a)**, **(b)**, or **(c)** in the space provided beside each graph below.

a) For every \$100 CDN you exchange at the bank, you receive \$94 USD in return. Compare dollars CDN versus USD.

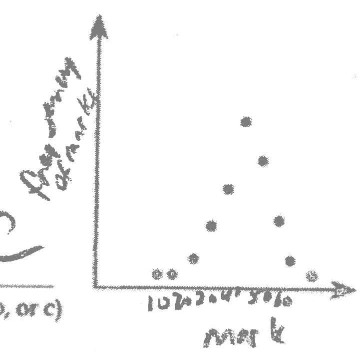
b) Jerome has two children, and then his children have two children each, and so on. Compare number of children versus time.

c) The class average was 85%. Only one person got 60%, and only four people got 92%. Compare mark percent versus frequency (ie *the number of people that got each mark*).

B
(a, b, or c)

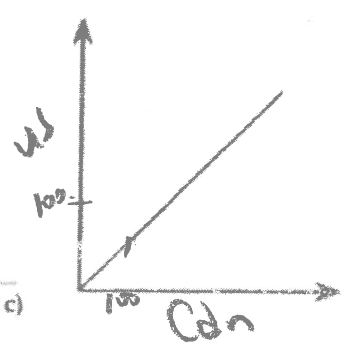


C
(a, b, or c)



most people got the middle mark. fewer got low + high marks

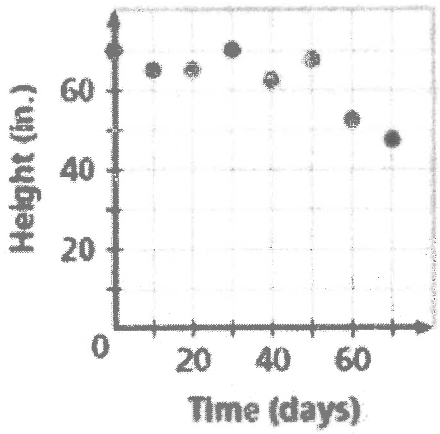
A
(a, b, or c)



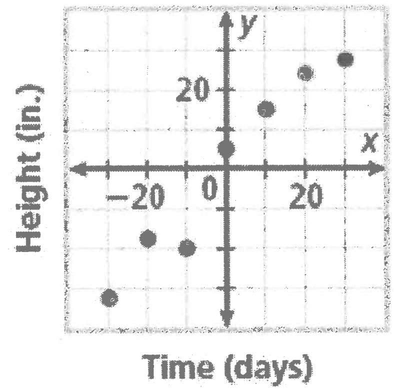
6a. Which graph best represents the number of days since a sunflower seed was planted and the height of the plant? C Why?

Seed has no height for a bit then gradually grows

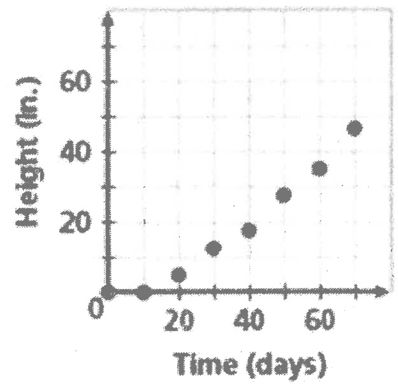
Graph A



Graph B



Graph C



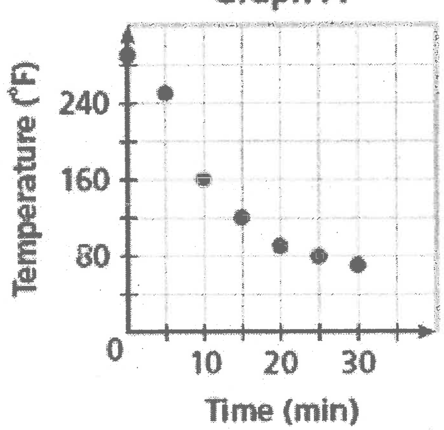
height starts at 70 in when seed planted?

Can't have neg. height neg days

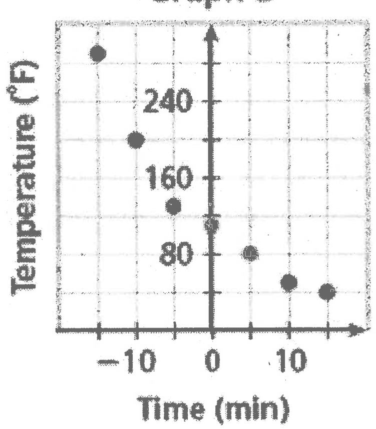
b.) Which graph best shows the relationship between the number of minutes since a pie was taken out of the oven and the temperature of the pie? A Explain.

Starts hot then cools

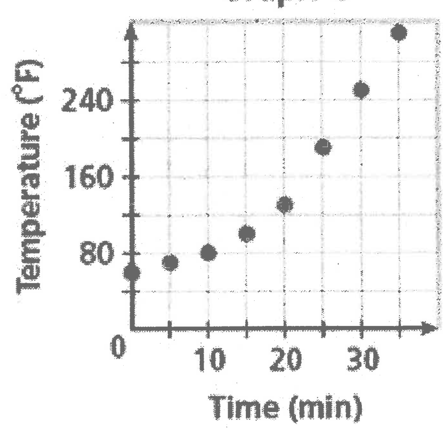
Graph A



Graph B



Graph C



can't have neg time.

doesn't get hotter when you remove from oven

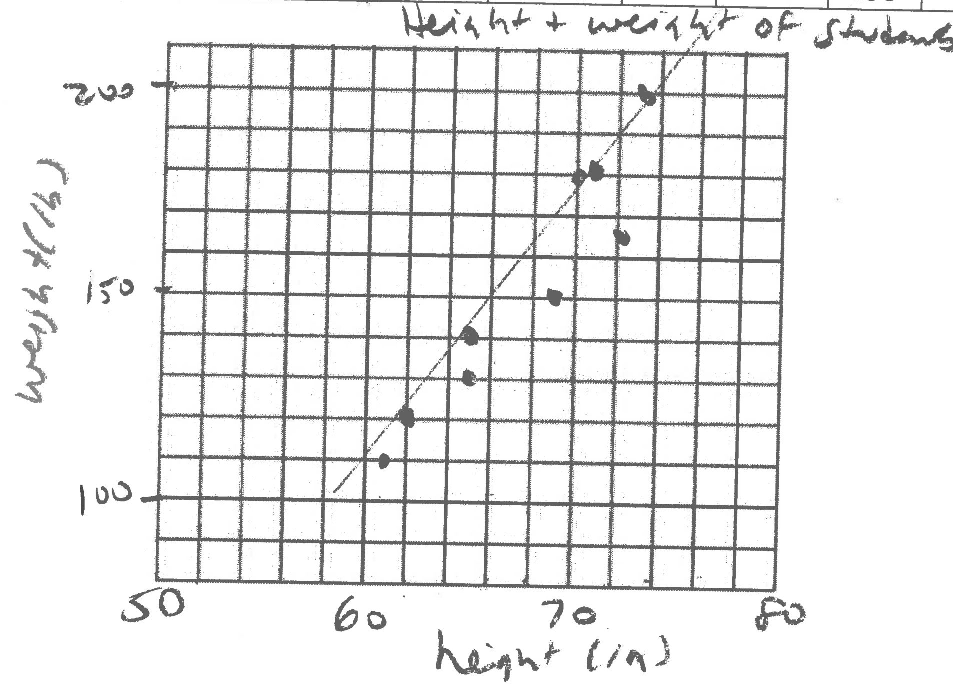
Lesson 2: Linear Patterns in Data

A ***linear relation*** can be represented as a **graph**, a **table of values** or as a **pattern**.

When plotted on a graph, a **linear relation** is a set of data that **looks as though a line could be drawn through it to represent the data.** *The data does not necessarily need to create a perfect line*, but overall, the points should form a straight line.

1. Try it: The following table of values shows the relationship between the height and weight of students in a class. Draw a scatterplot. (Follow all the steps – p. 6). Show height 50-80" and weight 80-200 lbs (start #s at 100).

Height inches	61	69	71	62	65	73	65	72	70
Weight pounds	110	150	180	120	140	200	130	165	180



Now that the data has been graphed, draw the ***line of best fit***, or the line drawn on a scatterplot that best represents the data on a scatter plot. **The line of best fit allows us to draw conclusions about the graph and to make predictions.** Not all of the points have to be on the line, but the line should be drawn through as many points as possible. **About half** the points not on the line should be **above** the line, and **about half** should be **below** the line.

Note: The line does not have to pass through the origin (point 0,0). Sometimes the line will not pass any of the points on the graph.

Note the characteristics of a **line of best fit on your resource sheet.**

When you draw the line of best fit, what trend do you see?

The taller a male student is, the more he weighs. This is a positive linear trend.

Linear Patterns – A **pattern** that represents a linear relation is known as an **arithmetic sequence** or a ***linear pattern***, such as: (**1, 5, 9, 13, 17**...) and (**14, 12, 10, 8, 6**...)

add 4 each time

subtract 2 each time

Linear patterns occur when a set of numbers is created by adding or subtracting the same number each time.

2. **Try it:** Identify the following patterns as **linear** or **non-linear**. If *linear*, say how you know.

a) 11, 19, 27, 35, 43... linear (add 8 each time)

+8 +8 +8 +8

b) 32, 16, 8, 4, 2... non-linear (subtract different # each time) ÷2 not linear

-16 -8 -4 -2

c) 1, 0, -1, 0, 1... ... Non linear (sometimes +1 sometimes -1)

d) 114, 101, 88, 75, 62... ... linear (subtract 13 each time)

-13 -13 -13

Linear Patterns in Table of Values

Patterns can be represented in tables of values. However, it is important to note that **both** the *x* and *y* values must increase or decrease by the same number **each time.**

Example:

In the example below, the ***x-axis*** or number of purchased apples **increases by five,** while the ***y-axis*** or number of free apples **increases by one.**

+5 +5 +5 +5

Number of apples purchased	5	10	15	20	25
Number of free apples	1	2	3	4	5

+1 +1 +1 +1

←*independent values*

←*dependent values*

By seeing if the x and y values increase or decrease by the same number each time, you are finding the rate of change. If the rate of change is the same for all possible sets of any two data points, then the relation is linear.

$$\text{Rate of change formula: Rate} = \frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$$

From the above example, compare 3 sets of data points:

(2,10) and (1,5) $\frac{2-1}{10-5} = \frac{1}{5}$ (3,15) and (4,20) $\frac{4-3}{20-15} = \frac{1}{5}$ and (4,20) and 5,25) $\frac{5-4}{25-20} = \frac{1}{5}$

The rate of change is the same for each pair of data sets is the same. **Therefore the relation is linear.**

Proportion:

There is another way to see if there is a linear pattern than by counting. Each category separately is a <u>linear pattern.</u> Therefore the entire set of data has a **linear relationship**. You can see if the pattern increases by the same **proportion** each time.

Number of apples purchased		Number of free apples
5	÷ 5	1
10	÷ 5	2
15	÷ 5	3
20	÷ 5	4
25	÷ 5	5

For every 5 apples you buy, you get 1 apple free. Therefore the number of apples purchases **<u>divided by</u>** 5 equals the number of free apples.

$\frac{5}{1} = \frac{10}{2} = \frac{15}{3} = \frac{20}{4} = \frac{25}{5}$ proportions the same.

You can predict how many free apples we could get if we bought 100 apples.

100 ÷ 5 = 20 free apples

3. Try it: Which of the following table of values represent a linear relation? How do you know?

a)

	Age (mo.)	Mass (kg)	
	0	3.0	+1
+1	1	4.0	+0.8
+1	2	4.8	+0.7
+1	3	5.5	
	4	6.2	

not linear
mass doesn't go up by same #.

b)

	Frame No.	Perimeter (unit)	
	1	4	+4
+1	2	8	+4
+1	3	12	+4
+1	4	16	
	5	20	
	6	24	

or 4 ÷ 1 = 1
8 ÷ 2 = 4

both sets of data increase by same #

<u>also</u> they increase by same proportion each time.

c) (You can use the counting method for the first 4 sets of data points, but because the data set doesn't jump by the same number each time, you will need to see if the proportions are the same.)

Number of floors in an apartment building	3 x12	4 x12	5 x12	6 x12	10 x12
Number of apartments in the building	36	48	60	72	120

$\frac{36}{3} = \frac{48}{4} = \frac{60}{5}$ yes. They increase by same proportion each time.

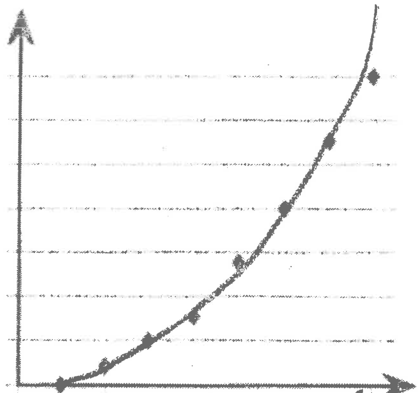
If there were 15 floors, predict how many apartments there would be in the building. 180 apts.

15 × 12 = 180

Linear Patterns Exercise

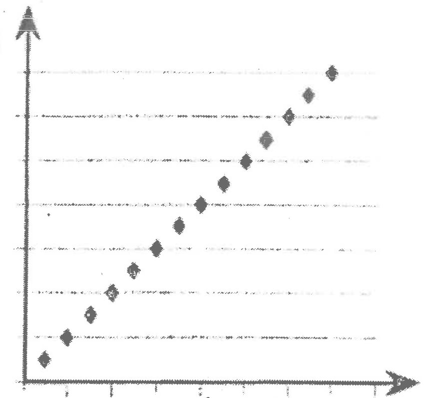
1. State whether the graph is linear, non-linear, or does not show a pattern.

a)



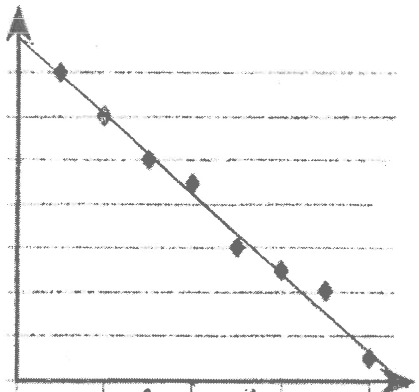
non-linear (curve)

b)



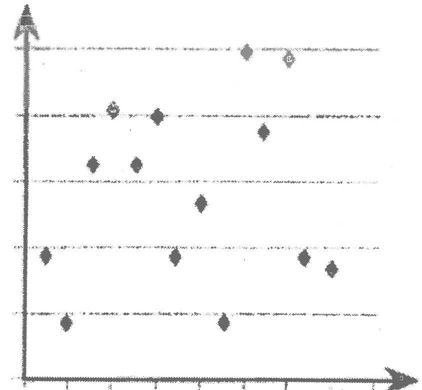
linear

c)



linear

d)



no pattern

2. State whether each pattern is linear or non-linear. If it is linear, what number do you add each time?

a) -4, -1, 2, 5, 8,... → linear/non-linear add 3 each time

+0 +1 +1 +1

b) 1, 1, 2, 3, 5,... → linear/non-linear don't add same # each time (add 2 #s to get next number is a pattern but not linear)

×4 ×4

c) 1,4,16,64, 256,... → linear/non-linear each number multiplied by 4 (must have same # added or subtr.)

d) 251, 272, 293, 314, 335,.. → linear/non-linear add 21 each time

3. Sort the following relations and patterns into linear and non- linear
(Hint: For the graphs, try to draw a line of best fit.)

a)

x	1	2	3	4
y	7	14	21	28

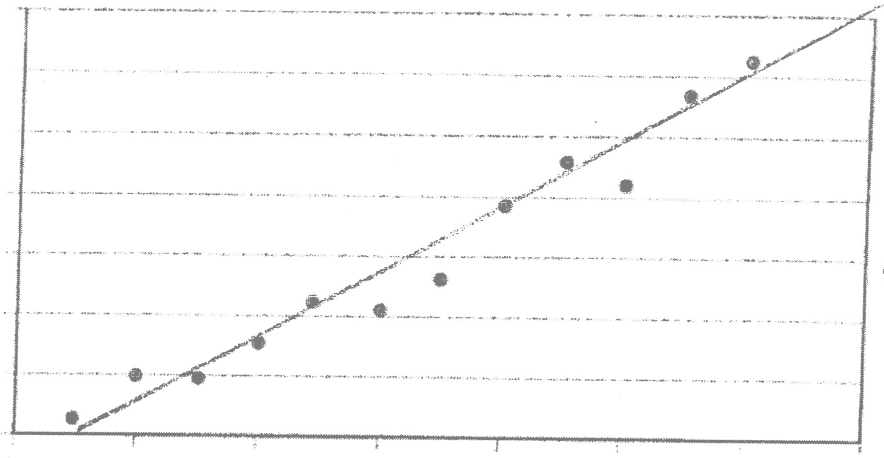
÷7 ÷7 ÷7 ÷7

linear
rate of change is 7 $\frac{28-21}{4-3} = 7$
proportions the same ($\div 7$)
$\frac{7}{1} = \frac{14}{2} = \frac{21}{3}$

b) –30, –19, –8, 3, 14, ... linear (add 11)
+11 +11 +11 +11

c) –2, –4, –6, –12, –14, ... not linear
–2 –2 –6 –2
difference not always 2

d)



linear (you can draw line of best fit)

e)

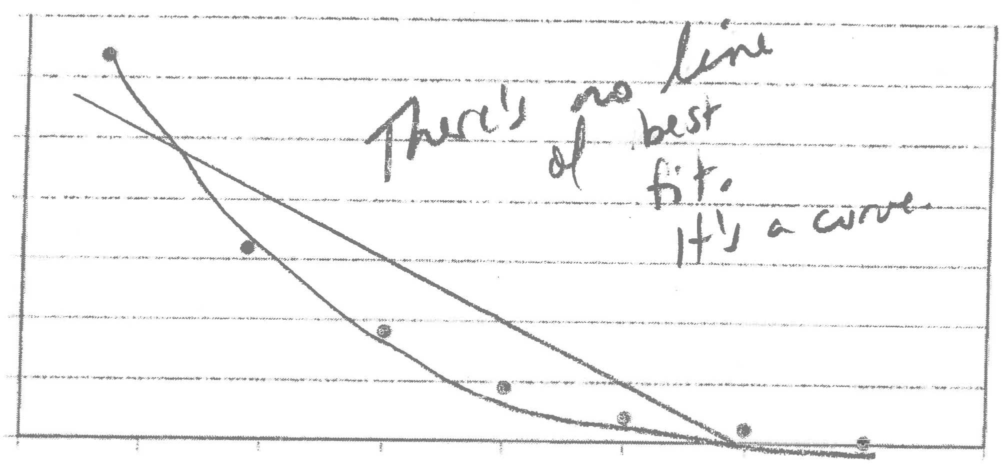
Distance (km)	1	2	5	6
Time (h)	6	12	30	36

×6 ×6

$\frac{6}{1} = \frac{12}{2} = \frac{30}{5} = \frac{36}{6}$

f) 6, 0, 2, 1, 0 ... not linear

g)



not linear.

+2 +2 +2

h)

Time (hours)	0	2	4	6
Temperature (°C)	25	23	21	16

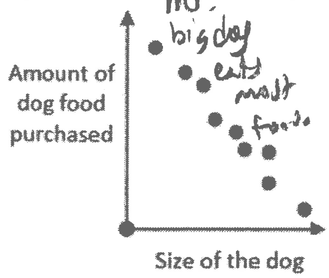
–2 –2 –5

rate of change $\frac{2}{-2} = -1$
except last one $\frac{2}{5}$

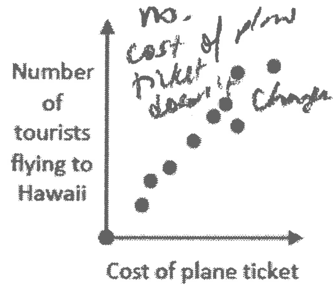
not linear

4 Which graph represents the correlation of its given situation correctly?

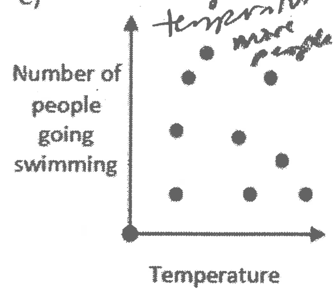
A)



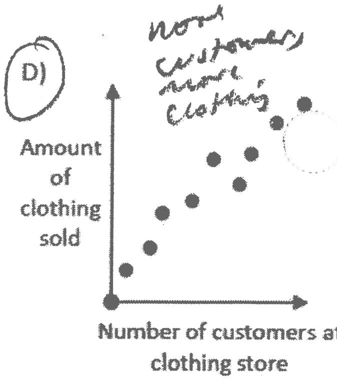
B)



C)



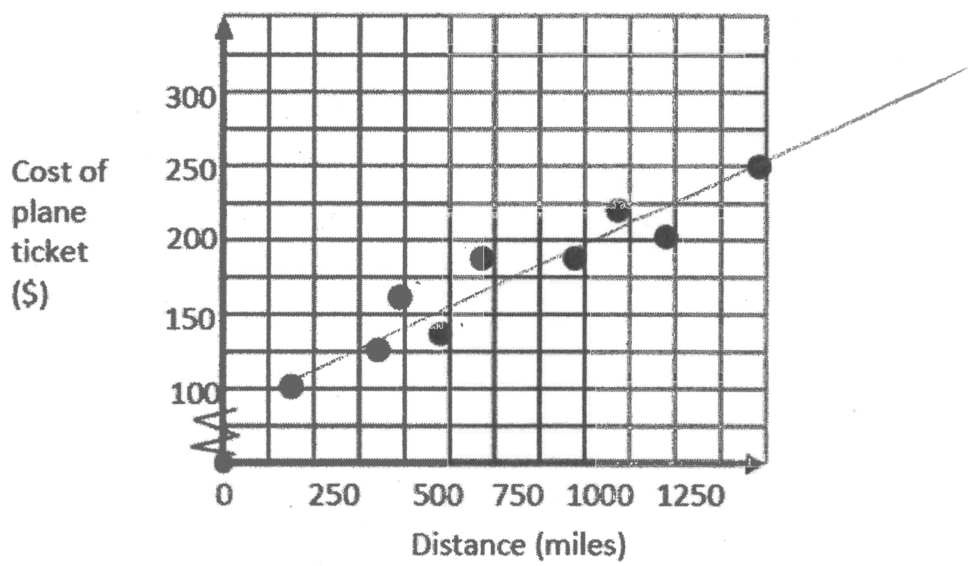
D)



5. The table below shows the cost of flying from San Francisco to various other cities in the United States There is a relationship between the distance you are flying and the cost of your plane ticket. The data from the table is represented on the scatter plot.

Draw a line of best fit. When you draw a line of best fit, what trend do you see? What kind of trend is this? The greater the distance, the more the plane ticket costs.

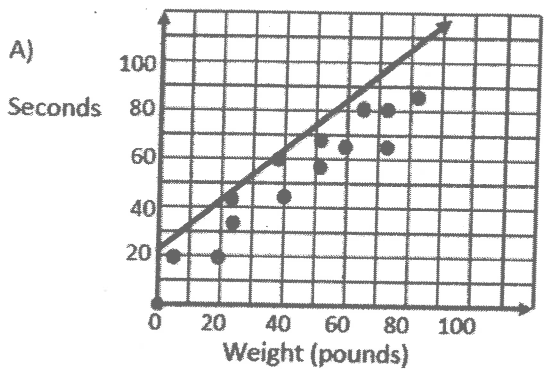
Distance(miles)	600	374	1,240	725	150	1,100	950	1,500	500
Cost of the plane ticket ($)	143	125	200	180	110	224	180	250	164



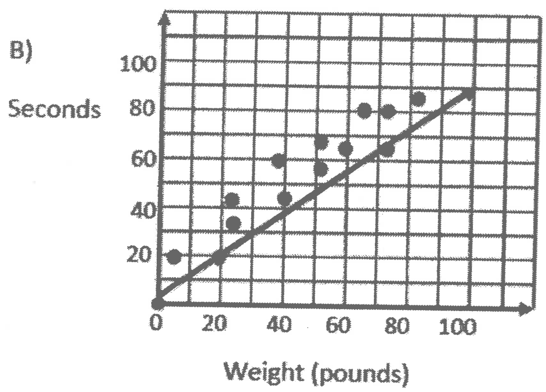
6. The graph shows the weights of dogs and the time it took the same dogs to complete an agility course in seconds.

Which shows the line of best fit for the data?

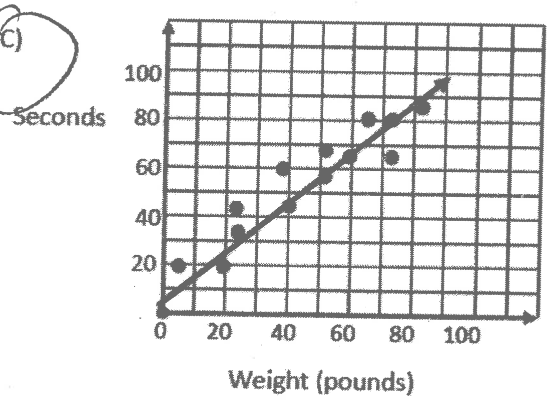
A)



B)

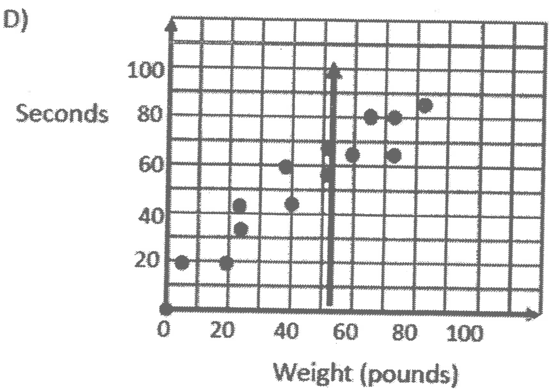


C)



about half above and half below

D)



Lesson 3: Equations of Linear Relations

An <u>equation</u> is a mathematical statement that describes a relation. The letters in the equation are the **variables**, and <u>as one variable changes, the other variable also changes</u>.

You can show a relation in <u>words</u>, as an <u>equation</u>, as a <u>table of values</u>, and as a <u>graph</u>. When you are given a linear relation represented in one of the above 4 ways, you can express it in any of the other ways.

Example: Emma is paid an hourly rate of $15. She worked the following hours: Mon: 6h, Tues: 8h, Wed: 0, Thurs: 4 h, Fri: 5 h.

a) Express the relation between daily gross pay and hours worked in **words**.

gross pay / equals / $15 / times / # hours worked

means 15 times h

b) Express the relation between the daily gross pay and hours worked as an **equation**. (Translate the words in (a) into variables and Mathematical symbols.) **Define your variables** (say what each variable represents).

$$p = 15h$$

means 15 times h

c) Use the equation to express the relation with a **table of values**. (The <u>independent variable</u> goes on the **left** and the <u>dependent variable</u> goes on the **right**. (*see page 3*). Put a **title** for each column with the variable and the word(s) that the variable represents. <u>If there is a unit, include it.</u>

Day	hours worked h	gross pay ($) p
Mon	6	90
Tues	8	120
Wed	0	0
Thurs	4	60
Fri	5	75

$p = 15h$

$p = 15(6) = 90$

$p = 15(8) = 120$

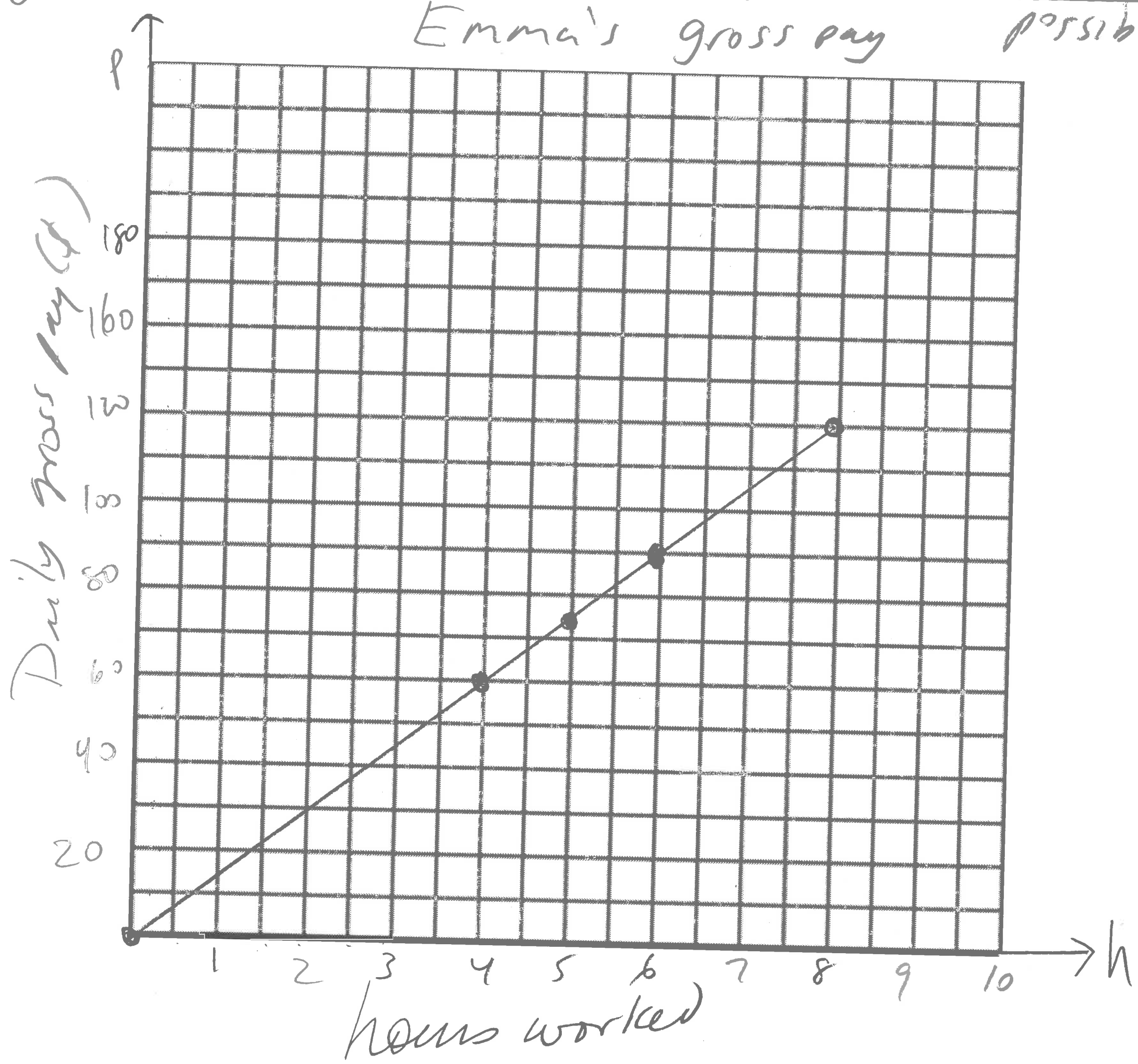
$p = 15(0) = 0$

$p = 15(4) = 60$

$p = 15(5) = 75$

d) Express the relation between daily gross pay and hours worked as a **graph**. (Remember all the steps to drawing a graph – p. 4) **Should you join the points? Why or why not?** If you join the points, use a ruler.

Yes – other amounts of hours and gross pay exist between points. Half hours possible



Linear Equation - Direct Variation

Direct variation is a simple mathematical relationship between two variables that can be expressed by an equation in which **one variable is equal to a constant** (a number) **times the other.** A linear relation of the form $\boxed{p = 15h}$ is called a ***direct variation***. We say "p **varies directly** as h" or "p is **directly proportional to** h". In this equation, 15 is the **constant of variation** or the **slope (m)**. Note that the *constant of variation* is simply the *hourly rate of pay.*

***15 is Constant of variation,** or **Slope** (m)* k

Graphs of direct variations have the following properties:

- **Straight** line
- Pass through the **origin**, or point (0, 0)
- **Increase in value** as you move right along the horizontal axis

A direct variation is represented by the equation:

Dependent variable = slope × independent variable or *y = mx*

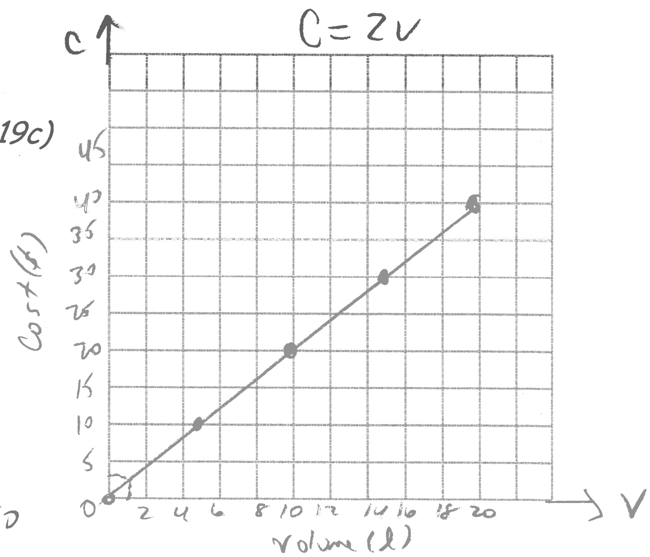
Put information about **direction variation** on your resource sheet.

Example: The **cost (c) of** bottled water in dollars **varies directly** with its **volume (v)** in litres. The **slope** *(constant of variation)* is 2.

a) Express this variation as an **equation.** $C = 2V$

b) Express this variation as a **graph. (First construct a table of values.** *Remember instructions for creating a table of values (p. 19c) and for creating a graph (p. 4).* **Should you connect the dots?**

volume (l) v	Cost ($) c	
0	0	C=2(0)
5	10	C=2(5)
10	20	C=2(10)
15	30	C=2(15)
20	40	C=2(20)



c) How much would 1 litre of bottled water cost? $2.50

The constant of variation or slope is 2. We can think of the constant of variation or slope in this example as representing the unit pricing (cost per unit).

Linear Equation - Partial Variation

What happens if the graph doesn't intersect or pass through the origin?

15h   +

Example: Emily is paid $15 per hour. In addition to her hourly rate, she receives a bonus of $50 each day for travel expenses. Express the relation between her daily gross pay (*p*) and her hours worked (*h*).

a) As an equation: $p = 15h + 50$

gross pay per day (p) equals $15 times the number of hours plus $50

b) With a **table of values**

hours worked h	gross pay ($) p
0	50
1	65
2	80
3	95

$p = 15(0) + 50 = 0 + 50$

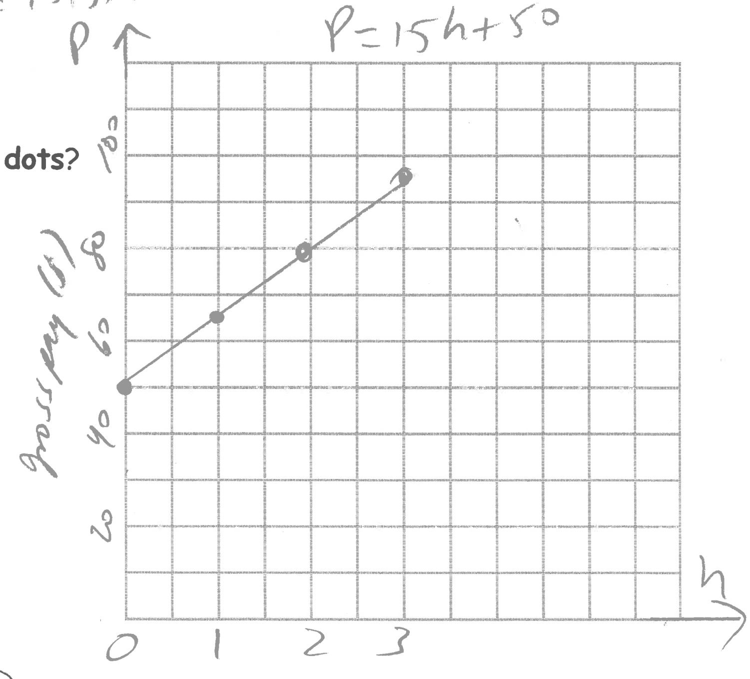
$p = 15(1) + 50 = 15 + 50$

$p = 15(2) + 50 = 30 + 50$

$p = 15(3) + 50 = 45 + 50$

c) As a **graph**. **Should you connect the dots?**

Yes. You can work $\frac{1}{2}$ hr $\frac{1}{4}$ hr. etc.



Linear relations of the form, $p = 15h + 50$, are known as ***partial variations***. The value **15** represents the **constant of variation**, k *(or slope, m)*. The value **50** represents the ***fixed value, F*** ***(or y-intercept)***.

slope m
constant k

fixed value F
y-intercept

Form of Partial Variation:

The fixed value is the value of the *dependent variable*, when the *independent variable has a value of 0*, (and is also known as the y-intercept *b*). The graph does ***not*** intersect the origin (0, 0).

A partial variation is represented by the equation:

	Dependent variable = k × independent variable + F
or	***Dependent variable = m × independent variable + b (in other words: y = mx + b)***

Try it: A car rental company charges $30 per day, plus 5 cents per km to rent a car.

a) Identify the type of variation: partial

b) Express this variation as an equation: $C = 30 + 0{,}05d$

(Choose the letters. Say what they represent.)

$C \rightarrow$ rental cost $\quad d \rightarrow$ distance driven (km)

$5¢ = \$0{,}05$

c) With a **table of values**. Use 0 km, 200 km, 400 km, 600 km

distance (km) d	Cost ($) C
0	30
200	40
400	50
600	60

$30 + 0{,}05(0) = 30$

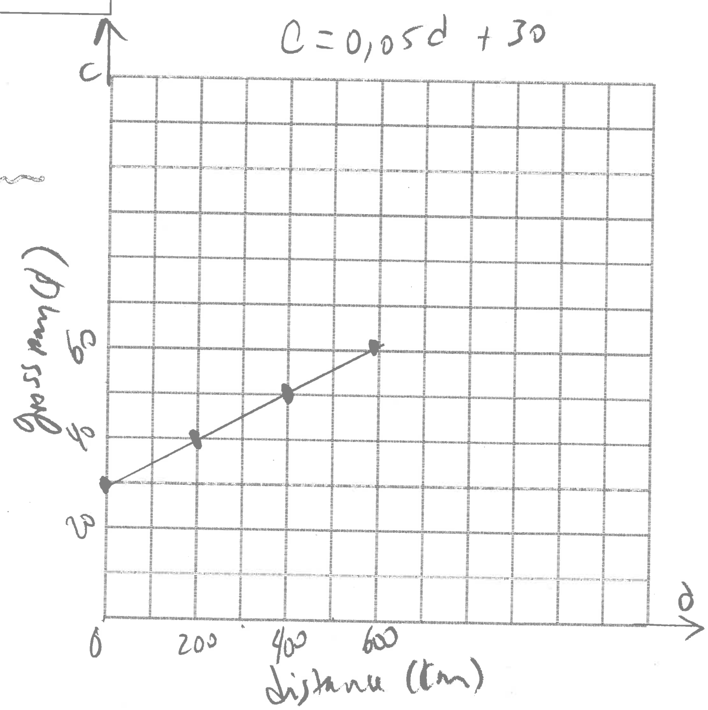
$30 + (0{,}05)(200) = 30 + 10$

$30 + (0{,}05)(400) = 30 + 20$

$30 + (0{,}05)(400) = 30 + 30$

d) As a **graph**

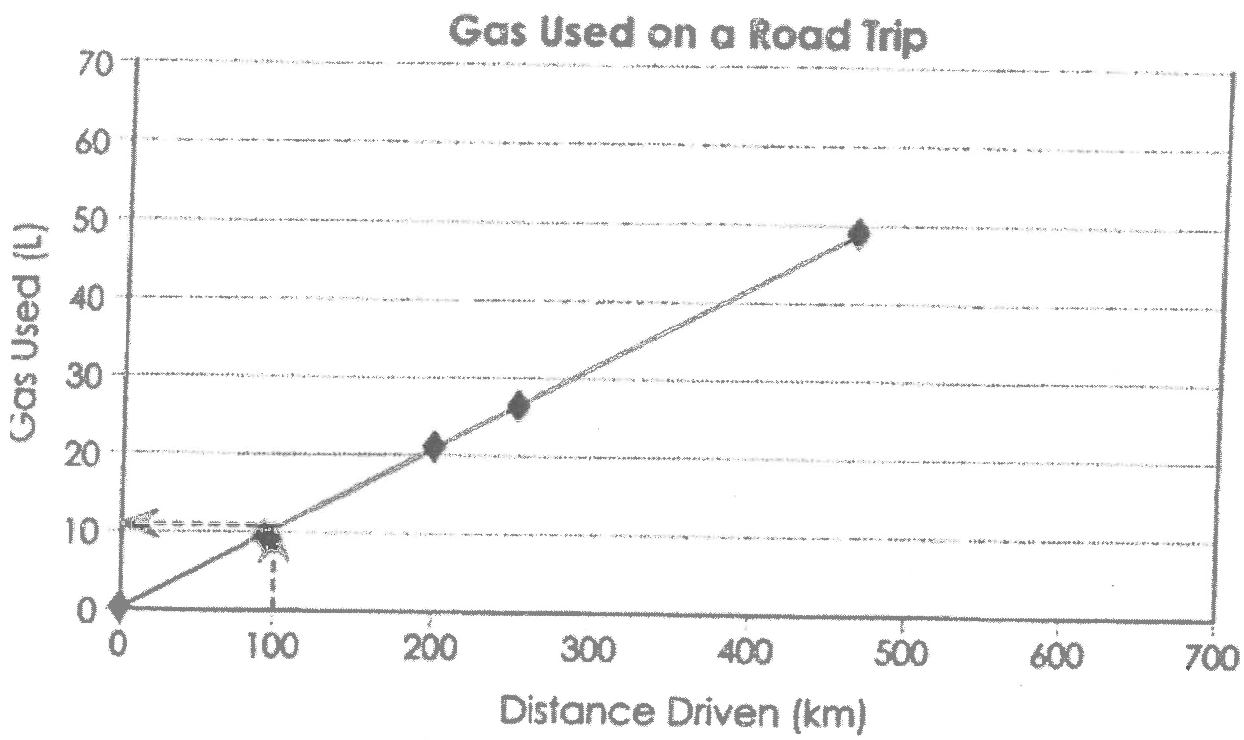
$C = 0{,}05d + 30$



join points
→ distances between 0 and 200 exist

Interpolation and Extrapolation:

EXAMPLE: The following graph displays the amount of gas used on a road trip. The car has a 60 L gas tank.



a) Use the graph to complete the following table of values (*hint: Use a ruler to connect the distance driven to the graph.. and then to connect that point on the graph horizontally to gas used.)* Finding the gas used for 300 km is called **INTERPOLATION** because you are estimating using the graph and because 300 litres is **between** other known values. *See page 2.*)

Distance driven (km)	100	300
Gas used (L)	11	31

b) How far could they drive before running out of gas? (hint: Use a ruler to extend the graph to just past 60 litres. Then use the same method as for (a). This is called **EXTRAPOLATION** because you are estimating a value using the graph but the value is BEYOND the end of the graph.) ~575 km

Equations of Linear Relations Exercise

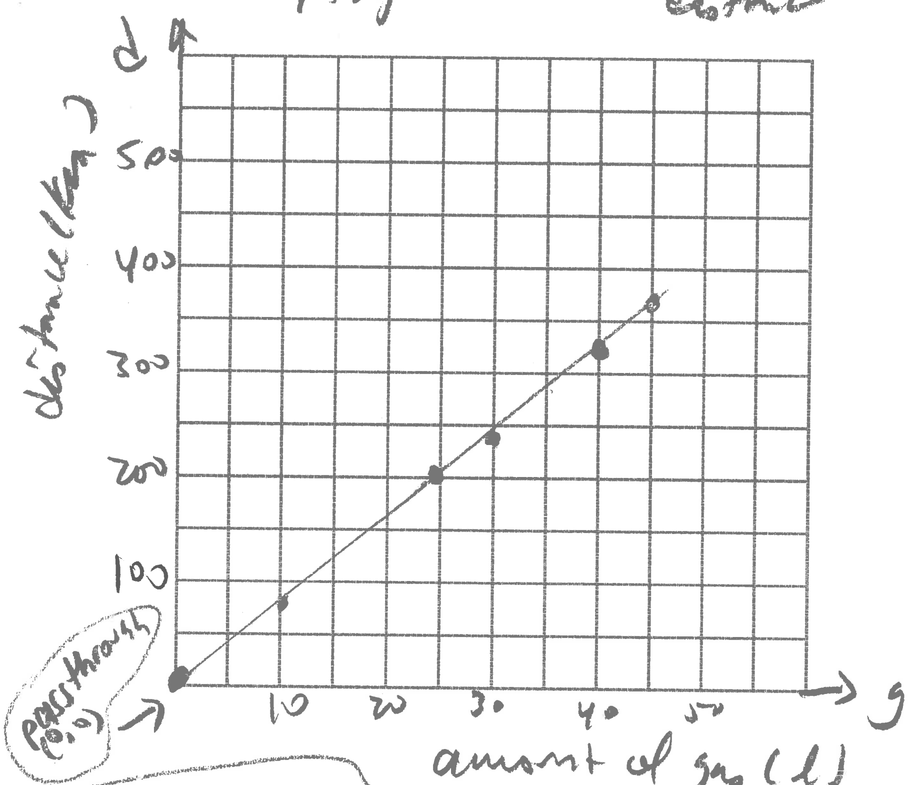
1. Under average road conditions, a vehicle can travel eight kilometres on one litre of gasoline. During the month of August, a vehicle uses the following amounts of gasoline: 30 litres, 45 litres, 10 litres, 40 litres, and 25 litres. Express the relation between the distance the vehicle travels and the amount of gasoline required to travel that distance.

a) in words   b) as an equation   c) with a table of values   d) as a graph

a) distance travelled equals 8 times # litres of gasoline

b) **equation:** $d = 8g$

d) August - Gasoline and distance   graph title



c)

gas(l) g	distance(km) d
30	240
45	360
10	80
40	320
25	200
0	0

$8(30) = 240$
$8(45) = 360$
$8(10) = 80$
$8(40) = 320$
$8(25) = 200$

axes labelled with words, units, numbers, letter

fill up space given

join points - ruler

equally spaced #s with consistent scale

2. The cost of building a highway is $500 000 for each kilometre. Us C to represent the cost, and n the number of kilometres.

a) Is this a direct or partial variation? (no fixed amount ; passes through (0,0))

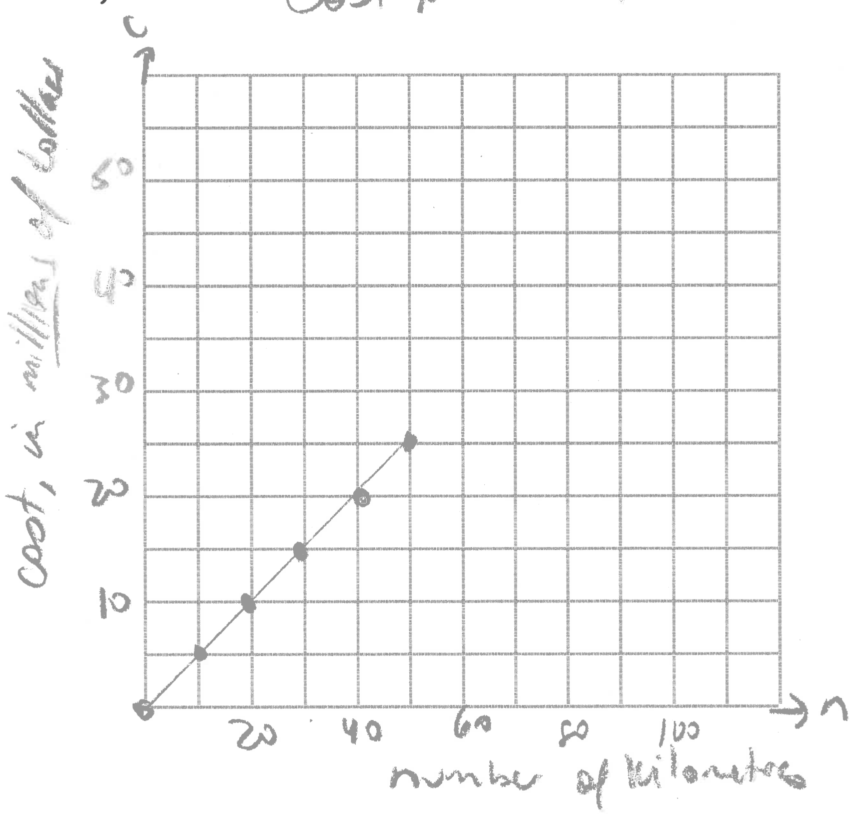
b) The constant of variation is: $k = 500\ 000$. Is there a fixed value? If so, what is it? no

c) Express the relation as an equation: $C = 500\ 000n$

d) Express the relation as a table of values. (Use 0 to 50 km, counting by 10 km). You could write out the millions for the cost or you could say "*cost in millions of dollars*".

e) Express the relation as a graph (choose appropriate scales for the coordinate axes).

e) Cost per km of building highway



d)

number of kilometres n	cost ($) C
0	0
10	5 000 000
20	10 000 000
30	15 000 000
40	20 000 000
50	25 000 000

Remember: graph title; axes labelled with words, units, numbers, letters; fill up the space given; join points with ruler; equally spaced numbers with consistent scale

3. Consider the variations d=10t and d = 10t + 40.

a) Which equation is direct? d=10t

b) Which equation is partial? 10= 10t+40

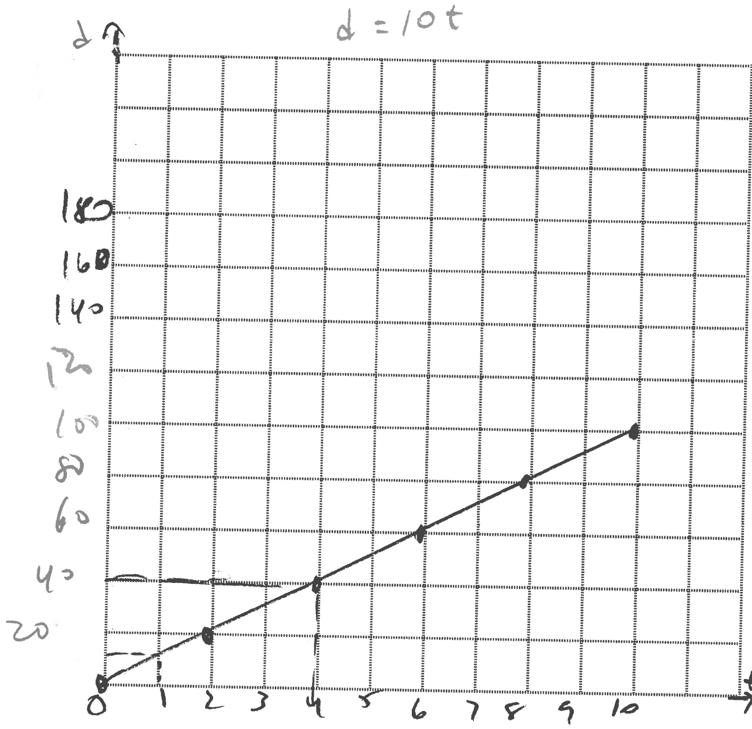
b) Create a table of values for each and then draw each of the variations as a graph on the given grid lines. Choose appropriate scales for the coordinate axes. use t= 0, 2, 4, 6, 8, 10

d=10t

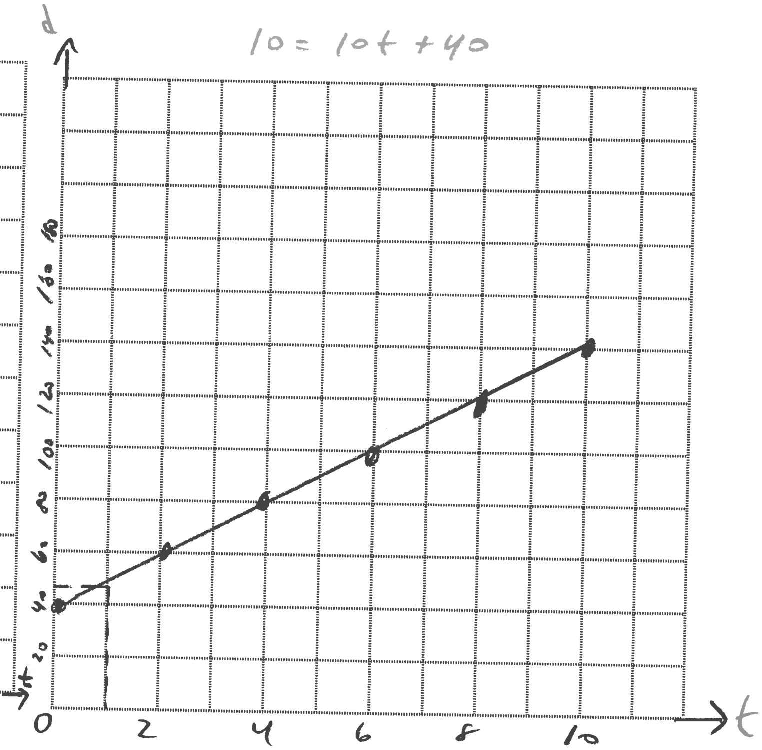
t	0	2	4	6	8	10
d	0	20	40	60	80	100

d=10t + 40

t	0	2	4	6	8	10
d	40	60	80	100	120	140



d = 10t



d = 10t + 40

c) find the value of d on both graphs when t = 1. 10 50

d) find the value of d on both graphs when t = 4. 40 80

e) One way the graphs are the same is: Same constant of variation (10) (same slope)

f) One way the graphs are different is: d=10t starts at 0,0. d=10t+40 starts at (0,40)

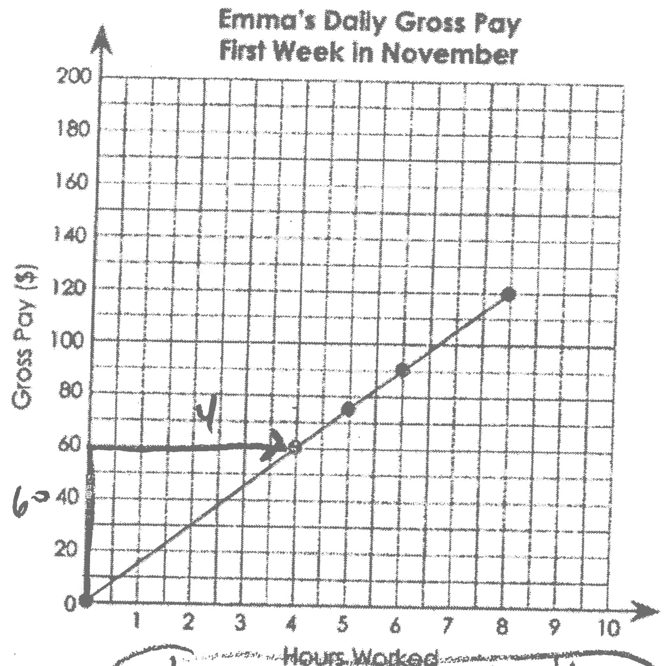
g) The **fixed value** (or the y-intercept) ~~represents~~ indicates: where graph crosses y-axis

32

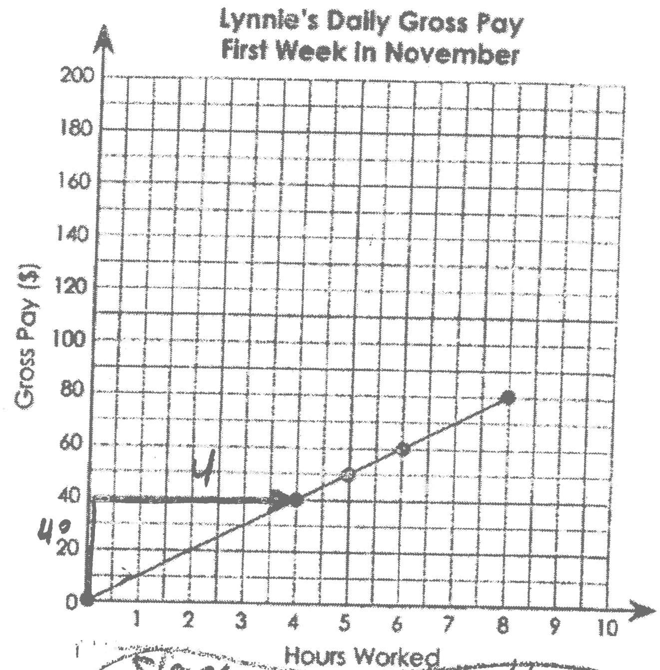
Lesson 4: The Slope of a Line

Take a look at the 2 graphs below. What is the same, and what is different?

Same: linear, start at (0,0)   different: Emma's graph steeper (steeper SLOPE)



slope = 15 ∴ $15/hr



slope = 10 ∴ $10/hr.

Calculating the slope of a line:

Method 1: Direct Variation: substitute into equation y = mx

EXAMPLE: What is the slope of Lynnie's graph?

1. Check whether the graph is a partial or direct variation. **It is a** direct **variation because it passes through the point** (0,0). Therefore, use the direct variation equation:

dependent variable = m × independent variable | **y = mx**

The **slope (m)** of a graph is also known as the ***constant of variation.***

2. Enter the coordinates into the equation **y = mx** to solve for the **slope, or m.** Choose a point on the graph, such as (4 hr, $40). Enter the coordinates into the equation **y = mx** *(without the units)*. We enter the *independent variable* **4** for **x** and the *dependent variable* $40 for **y**. When we divide both sides by 4 we see that the **slope (m)** is 10.

$$y = mx \qquad \frac{40}{4} = \frac{m(4)}{4} \qquad m = 10$$

$y = mx$, $40 = m(4)$ → $\frac{40}{4} = \frac{4m}{4}$ → $10 = m$

3. **To check if this is correct**, start at the origin. Move up to $10 on the y-axis and then right one hour on the x-axis. If you are on the line, the slope is correct.

$\frac{rise}{run} = \frac{40}{4} = 10$

Slope is a ratio, or comparison of the "*change in the dependent variable*" to the "*change in the independent variable*". Slope is the number that describes how far a line moves **vertically**, compared to how far it moves **horizontally**. It describes the **rate** (how fast) at which a line *falls* or *climbs*. *Slope can be expressed as an integer (ex.* ***15 or -15****), as a simplified fraction (ex.* $\frac{5}{2}$*), or as a decimal (ex.* ***2.5****).*

Method 2: Use a slope formula.

There is another way to calculate slope, which works for any linear relation, not just direct variation.

EXAMPLE: Calculate the slope of Emma's and Lynnie's pay graphs.

Step 1: Chose 2 points on Emma's line graph. It is most convenient to choose points that are at the intersections (or on the lines). Refer to the chosen points as Point A and Point B. So, Point A represents a gross pay of \$60 and 4 hours worked (4,60) while Point B represents a gross pay of \$120 and 8 hours worked (8,120).

(4,60) (8,120)
x y x y

Step 2: Find the vertical change (rise). Calculate the change in gross pay from Point A to Point B. *Change in pay = \$120 - \$60, which is \$60.*

Step 3: Find the horizontal change (run). Calculate the change in hours worked from Point A to Point B. *Change in hours worked = 8 hours - 4 hours, which is* ***4 hours****.*

Step 4: Divide. 60 ÷ 4 = 15.

$$slope\ (m) = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$m = \frac{\text{change in gross pay}}{\text{change in hours}}$$

$$m = \frac{120-60}{8-4}$$

$$m = 15$$

The slope is 15, so Emma's wage is \$15 per hour.

Use a formula to calculate the slope of the line.

(Do not include the units in the formula.)

$$\text{slope } (rate) = \frac{\text{change in dependent variable } (vertical)}{\text{change in independent variable } (horizontal)}$$ or $$slope = \frac{\text{vertical change}}{\text{horizontal change}}$$ or $$slope = \frac{rise}{run}$$

Because the dependent variable is on the vertical or y-axis, and the independent variable is on the horizontal or x-axis, the formula can also be written:

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

The 4 formulas given for slope are all the same formula, just written in different forms.
You need to use the one easiest for you (and the one that works for the situation.)

1. **Try it:** Calculate the slope of ~~Lynnie~~ Emma's pay graph in the same way you calculated ~~Emma~~ Lynnie's slope.

$\frac{\text{rise}}{\text{run}} = \frac{60}{4} = 15$

or

or $(4,60)\ (6,90)$

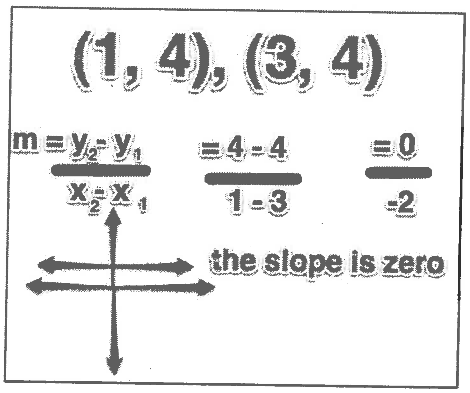
$\frac{90-60}{6-4} = \frac{30}{2} = 15$ ← slope formula or $\frac{\text{change in } y}{\text{change in } x}$

∴ \$15 an hour.

Emma has the larger hourly rate, \$15.00 per hour, as compared to Lynnie's hourly rate, \$10.00 per hour. Therefore, Emma has the *greater value of the slope*, which results in a **steeper graph of the relation. In general, the <u>greater the value of the slope</u>, the <u>steeper the graph of the relation</u>. This is true of any linear relation.**

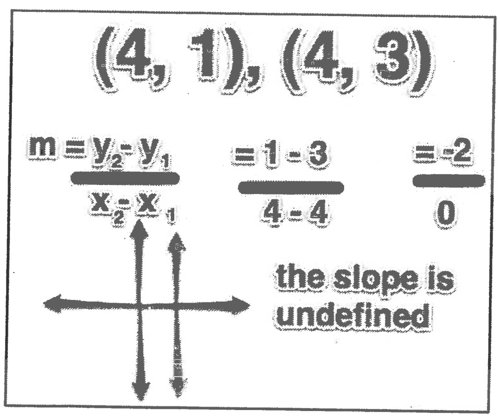
Two special Cases of Slope

- ***Special Case #1:*** ***The slope of a horizontal line is always zero. This is because the change in the dependent variable or ($y_2 - y_1$) equals zero. A value of zero in the numerator always results in a slope that is zero.***



Put the the pair of coordinates (1, 4) and (3, 4), into the slope formula. That gives you four minus four divided by one minus three, which reduces to zero over negative two. That means you have a slope of zero. **Anytime you have a line with a slope of zero, you know that line will be horizontal.**

- ***Special Case #2:*** ***The slope of a vertical line is always undefined. This is because the change in the independent variable or ($x_2 - x_1$) equals zero. A value of zero in the denominator always results in an undefined slope.***



Put the pair of coordinates (4, 1) and (4, 3), into the slope formula. That gives you one minus three divided by four minus four, which reduces to negative two over zero. <u>Since you can't divide a number by zero</u>, that means you have an undefined slope. Anytime you have a line with an undefined slope, you know that line will be vertical.

Include the description of slope, the formulas, and special cases in your resource sheet.

Example 1

A theatre owner was interested in the relationship between time and the number of people in a movie. Throughout the 2.5-hour movie, not a single person enters or leaves the theatre. There were 80 people watching a movie.

a) The independent variable is time.

The dependent variable is number of people.

b) Express, in words, the relationship between time and number of people in theatre during the movie.

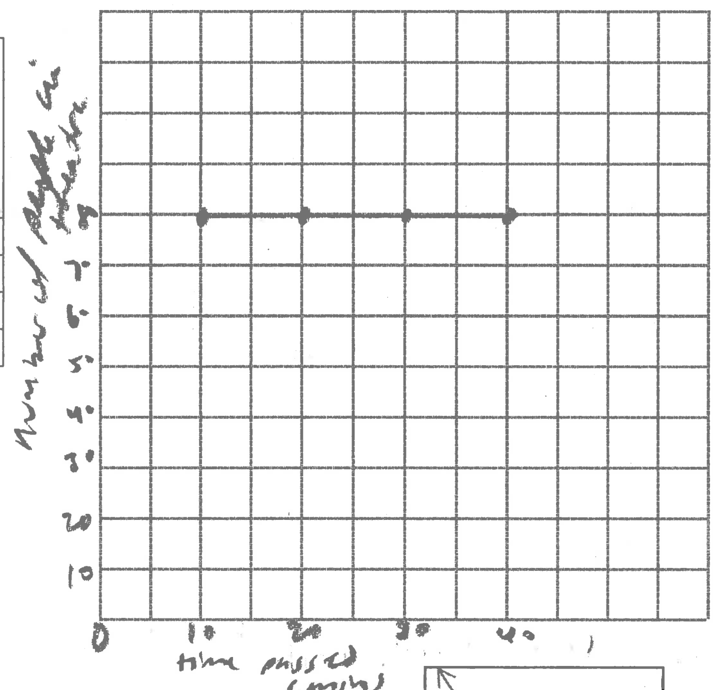
There are always 80 people in the theatre, no matter the time.

c) Express, as an equation, the relationship between time and number of people in theatre during the movie.

Use t for time and p for the number of people. $p = 80$

d) Draw a graph of the relation.

time passed (min)	number of people in theatre
10	80
20	80
30	80
40	80



e) Calculate the slope of the graph.

$\frac{rise}{run} = \frac{0}{10} = 0$

(10, 80)
(20, 80)

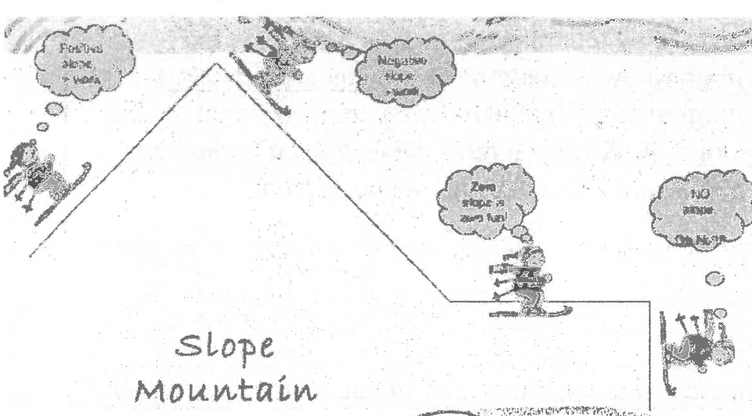
or $\frac{80-80}{20-10} = \frac{0}{10} = 0$

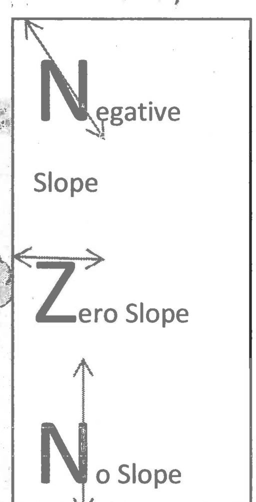
$m \; \frac{y_2 - y_1}{x_2 - x_1}$

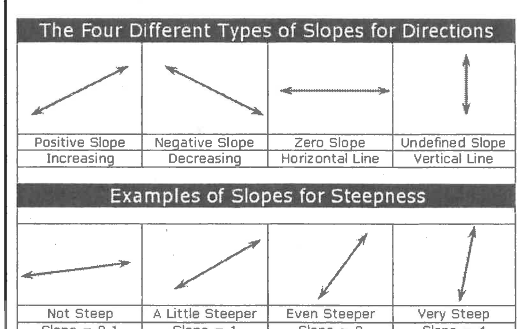
or

$\frac{\text{change in } y}{\text{change in } x}$

0 people per minute left or entered the theatre.







Special case #1 horizontal line slope is always zero.

Use a Graph to find Equation of Line and to Calculate Slope

EXAMPLE : Jayesh is saving his money and is interested in tracking the value of his investment over time. His principal amount is $200. Each year, his investment increases $15 in value.

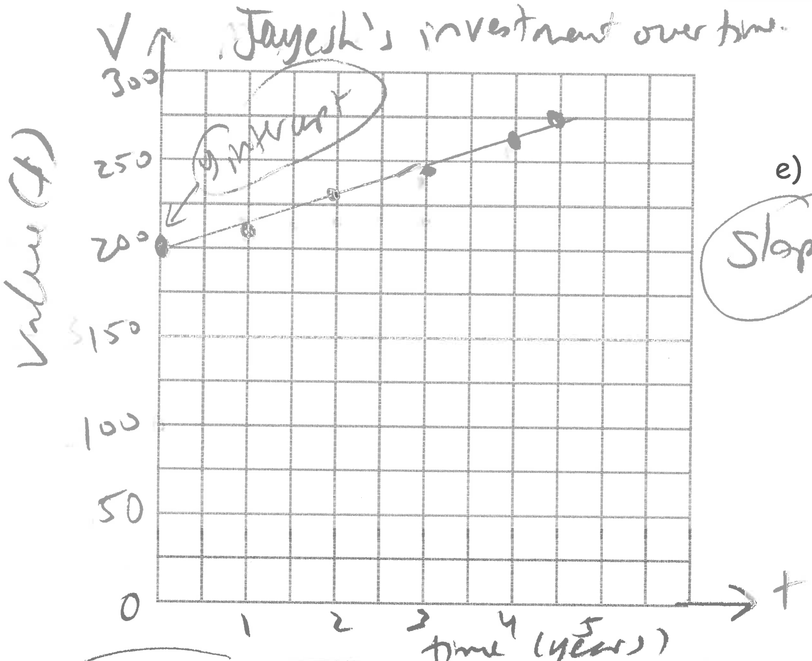
a) Is this relation a **direct** or **partial variation**? It starts with more than 0.

b) Express, in **words**, the relation between the value (v) of his investment, and time (t).
value equals $15 times # of years + initial investment of $200.

c) Express as an **equation**, the relation between the value (v), and time (t). $V = 15t + 200$

d) Draw a **graph** of the equation for the first 5 years of the investment. (First make a table of values.)

time (years) t	0	1	2	3	4	5
Value of investment ($) V	200	215	230	245	260	275



$V = 15t + 200$
slope (↑ 15), y-intercept (↑ 200)

e) Calculate the slope of the graph.

$$\text{Slope} = \frac{\text{change in dep}}{\text{change in ind.}}$$

$$= \frac{215-200}{1-0}$$

$$= \frac{15}{1}$$

$$= 15$$

$15 dollars per year.

NOTE: The slope (15) is the **same as** **the constant of variation** in the equation. This is true for all linear equations (direct or partial). **The equation for an oblique line** (not vertical or horizontal) can now be written:

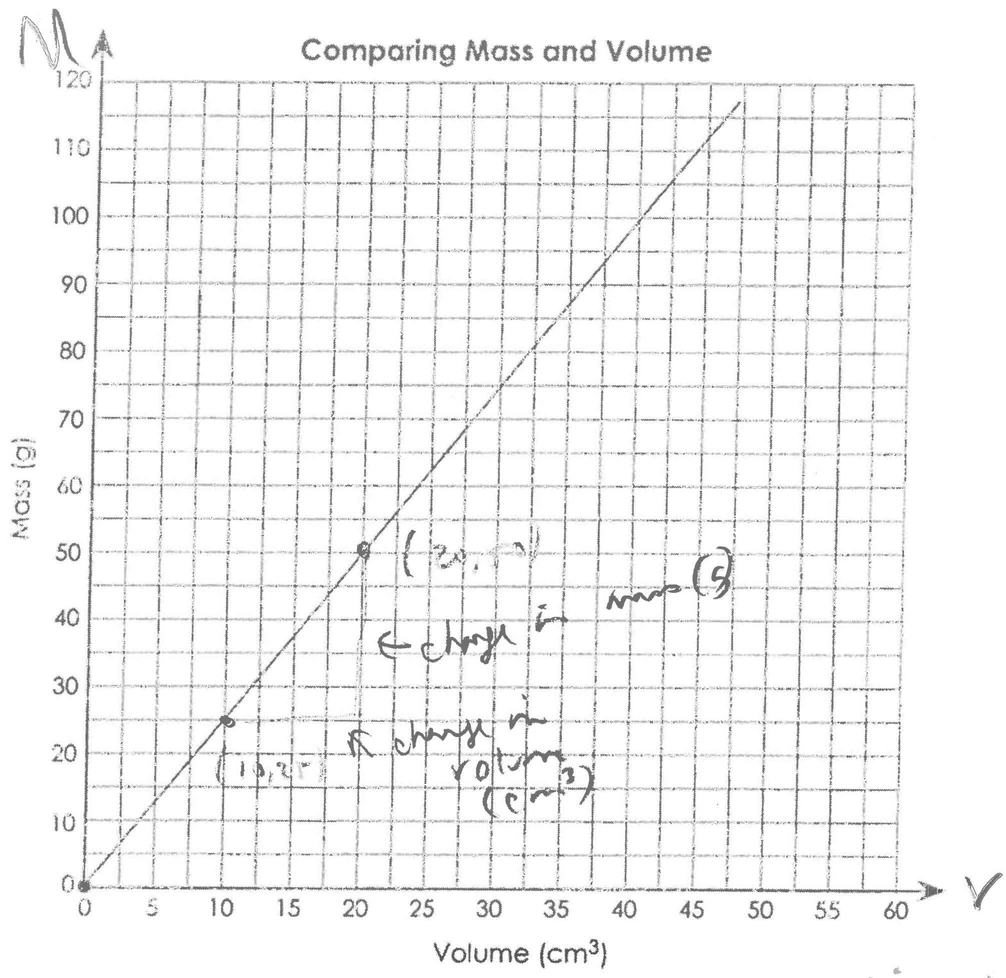
Partial variation: $\boxed{\textit{Dependent variable = slope} \times \textit{independent variable + fixed value}}$ ✱

*For a direct variation, the **fixed value is zero** and is not included.*

Direct variation: $\boxed{\textit{Dependent variable = slope} \times \textit{independent variable}}$ ✱

Write these general formulas on your resource sheet.

2. **Try it:** A student finds the volume and mass of several samples of a substance, and expresses the relation as the following line graph.



a) Calculate the slope of the line. (Locate 2 points on graph line where the grid lines cross. Write their coordinates

$$\frac{\text{change in dep. var}}{\text{change in ind. var.}} \qquad \frac{y_2 - y_1}{x_2 - x_1} \qquad \frac{50-25}{20-10} = \frac{25}{10} = 2.5$$

b) What does the slope of the line represent? $2.5\ g/cm^3$

rate of change of mass per unit of volume

c) Direct or partial variation? How do you know?
Direct because it crosses through (0,0)

c) Express the relation represented by the line graphs as an equation. $M = 2.5v$

Label the graph with letters for the variables - M for mass and V for volume

Hint extend lines to see if they cross through (0,0) to know if they are $y = mx$ or $y = mx + b$

Slope = rise/run = $\frac{y_2 - y_1}{x_2 - x_1}$

Examples: Slopes & Equations

note the scales!

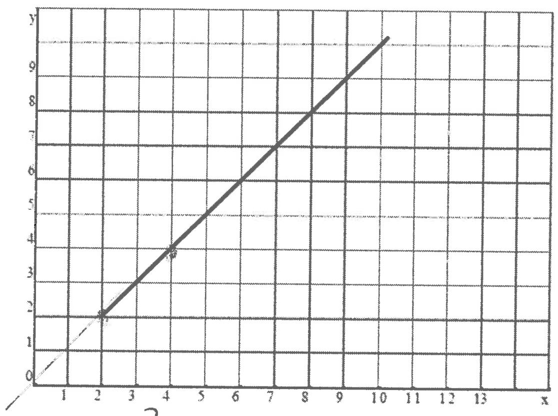
Name: or $y = mx + b$

Equation → $y = mx$

1. Determine the ***slopes*** and ***equations*** of the following lines:

a)

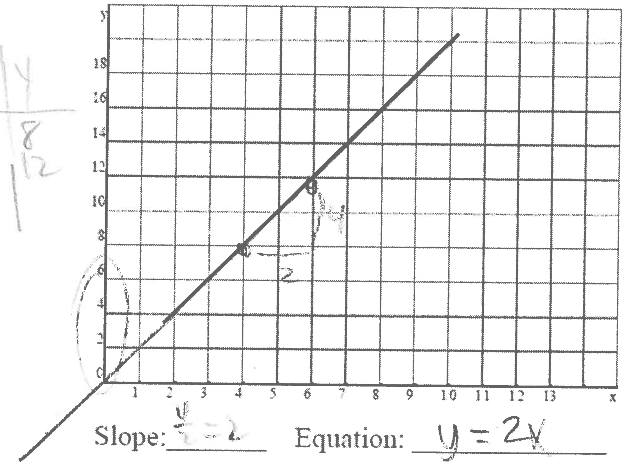
x	y
2	2
4	4



Slope: $\frac{2}{2} = 1$ Equation: $y = x$

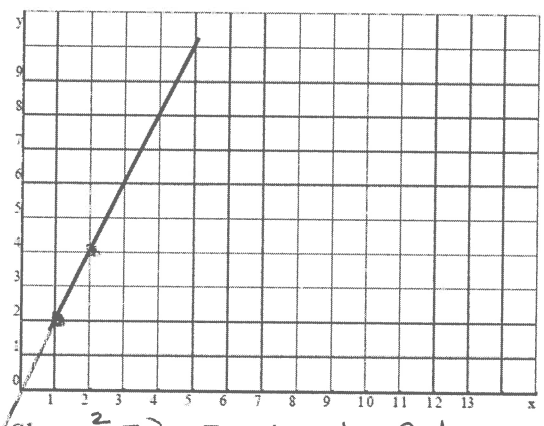
b)

x	y
4	8
6	12



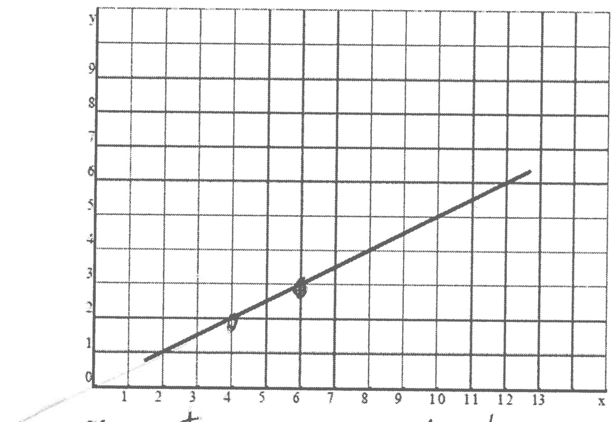
Slope: $\frac{4}{2} = 2$ Equation: $y = 2x$

c)



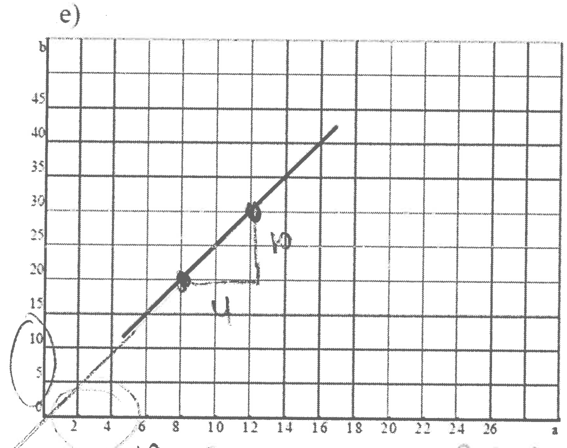
Slope: $\frac{2}{1} = 2$ Equation: $y = 2x$

d)



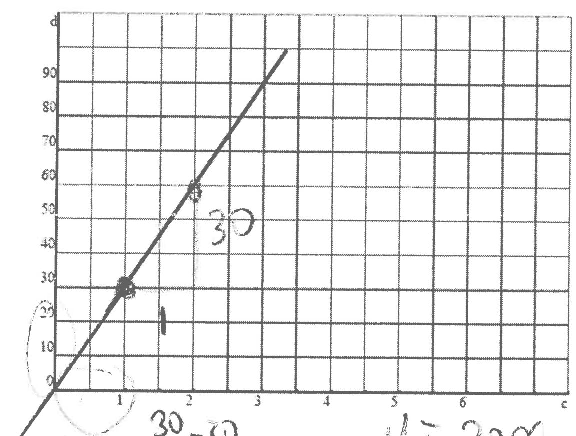
Slope: $\frac{1}{2}$ Equation: $y = \frac{1}{2}x$

e)



Slope: $\frac{10}{4} = \frac{5}{2}$ Equation: $y = 2.5x$

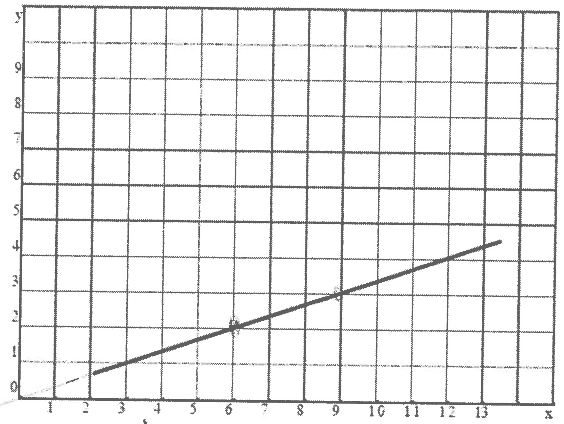
f)



Slope: $\frac{30}{1} = 30$ Equation: $y = 30x$

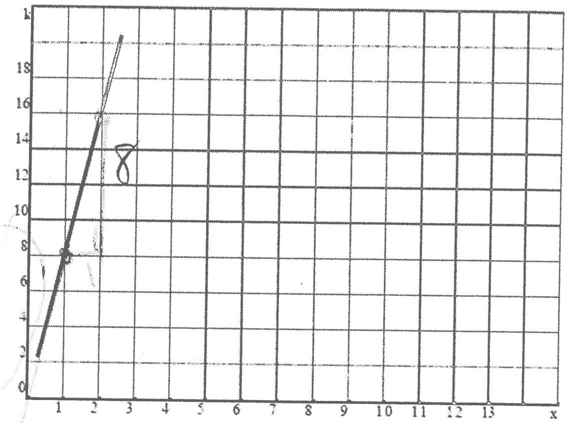
2. Determine the ***slopes*** and ***equations*** of the following lines:

a)



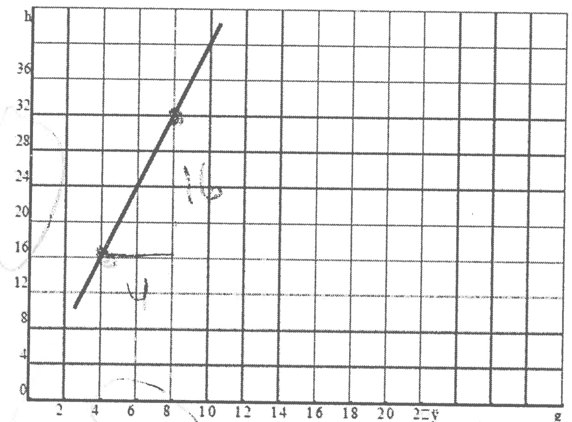
Slope: $\frac{1}{4}$ Equation: $y = \frac{1}{3}x$

b)



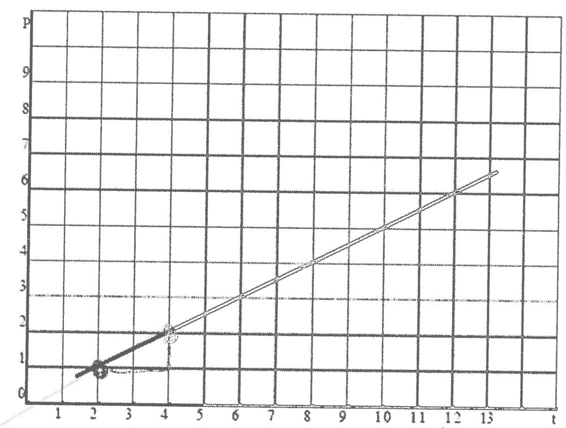
Slope: 8 Equation: $y = 8x$

c)



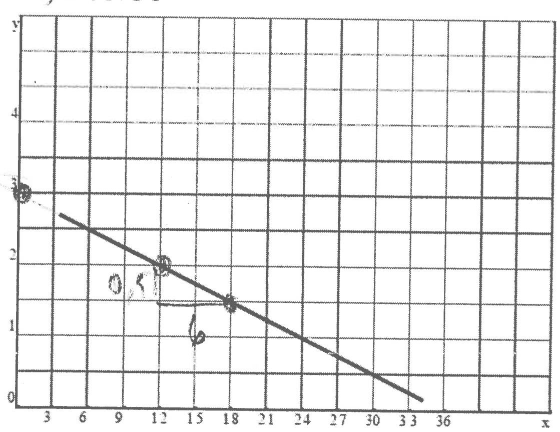
Slope: $\frac{16}{4} = 4$ Equation: $y = 4x$

d)



Slope: $\frac{1}{2}$ Equation: $y = \frac{1}{2}x$

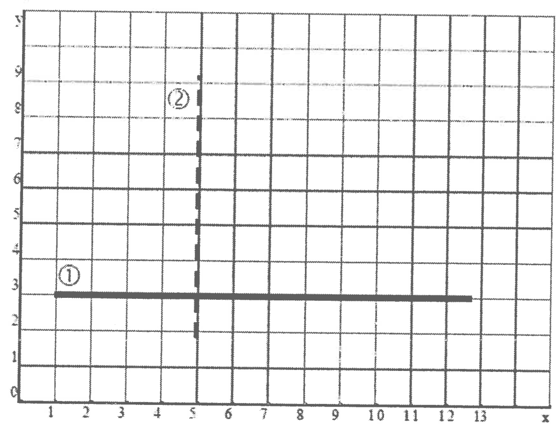
e) BONUS



Slope: $\frac{0.5}{6} = \frac{1}{12}$ Equation: $y = \frac{1}{12}x + 3$

$b = 3$

f)



① Slope: 0 Equation: $y = 3$

② Slope: undefined Equation: $x = 5$

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Use Table of Values to Calculate Slope and to Write Equation:

As well as using a graph to calculate slope, you can use a table of values to calculate the slope of a linear relation, and to then write the equation. **Slope is constant in all linear equations.** You must be sure the equation is linear, and then the slope will be the same each time.

EXAMPLE: A fitness club charges $50 to join, plus a monthly membership fee of $35. The table of values for some of the standard times is:

Number of Months of Membership m	3	6 ×43.3	12	18	← ricks
Total Cost ($) C	155	260	470	680	

a) is it linear? $\frac{155}{3} = 51.\overline{6}$

a) Calculate the slope of the data. (*Choose two data pairs to use. Calculate change in dependent variable; change in dependent variable; then divide.)*

To make sure the relation is linear, find slope with different pairs of data.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{\text{change in } y}{\text{change in } x}$

$= \frac{260-155}{6-3}$

$= \frac{105}{3}$

$= 35$

$\frac{680-470}{18-12}$

$= \frac{210}{6}$

$= 35$

$\frac{470-260}{12-6}$

$= \frac{210}{6}$

$= 35$

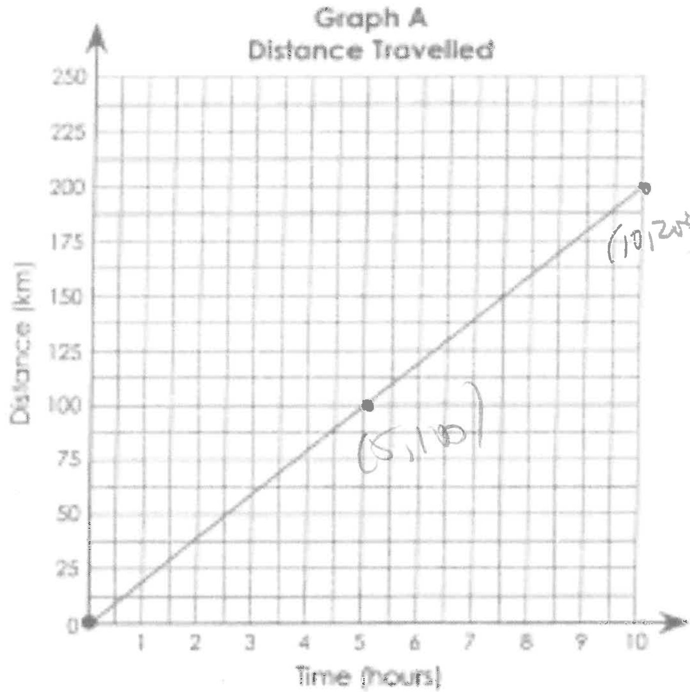
b) Express the relation as an equation. (*First determine whether the graph is a partial or direct variation.)* $C = 35m + 50$

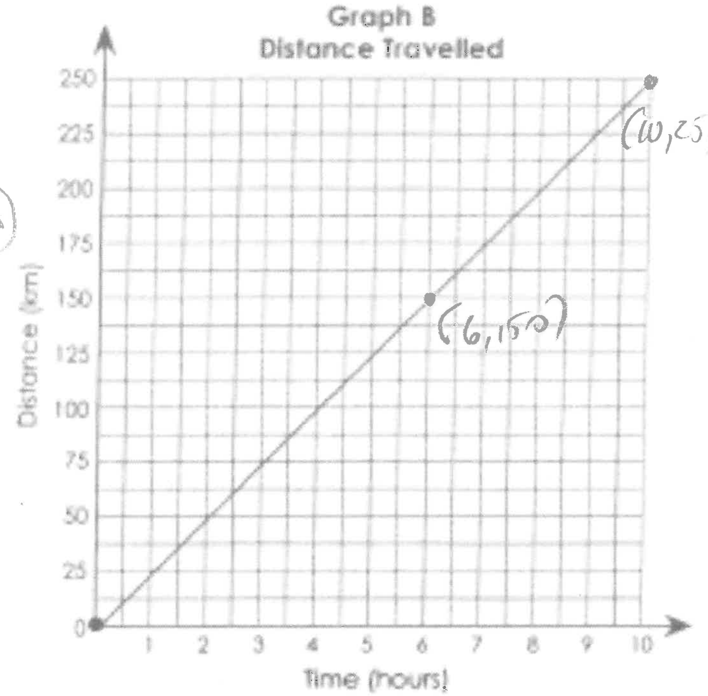
partial → plus membership fee

include steps in your resource sheet

Slope of a Line Exercise

1.





a) What does the slope of each line graph represent? Speed (km/h) each vechicle travelled

b) By observation, which line graph is steeper? B Explain what that means about the slope.
vehicle in graph B faster

c) Using the formula for slope, determine the slope of each of the line graphs.

A $\frac{200-100}{10-5}$ $= \frac{100}{5}$ $= 20$ km/h

B $\frac{250-150}{10-6}$ $= \frac{100}{4}$ $= 25$ km/h

d) How do the slopes of the line graphs compare to their steepness?
larger slope represents steeper line

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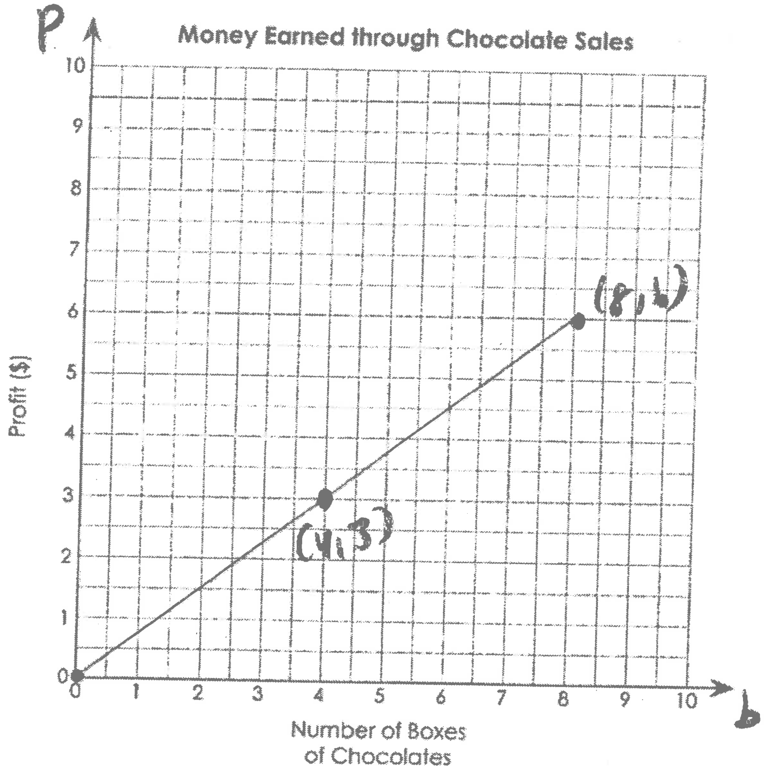
2. All horizontal lines have a slope of zero. In your own words, explain why.

Change in dependent variable (vertical change of graph) is zero. Zero divided by any number equals zero. So the slope is zero.

3. The following line graph represents the relation between the number of boxes of chocolates a student sells for a school fundraiser and the profit he earns for the school.



a) The **independent variable** is # boxes of chocolate and the **dependent variable** is profit.

b) Determine the **slope** of the line graph.

$$\frac{6-3}{8-4} = \frac{3}{4} = \$0,75 \text{ per box}$$

c) Express the relation represented by the line graph as an equation. $p = 0,75b$

d) What does the slope of the line graph represent? profit earned per box of chocolate sold, in dollars per box.

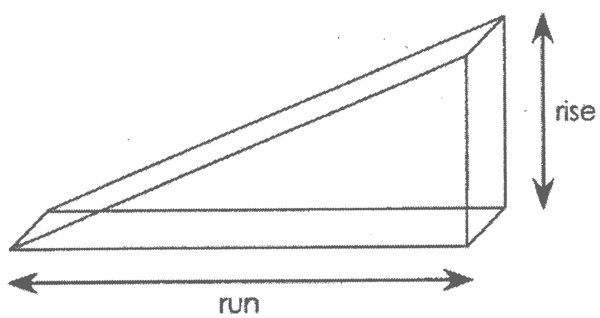
Lesson #5: Slopes of Objects:

In this lesson, you will

- Describe slope as '**rise over run**'
- Determine whether an object has a constant slope
- Look at the **safety implications** of slope
- Study the relationship between **slope** and **angle of elevation**

Slope of Objects - Slope equals rise over run

When we know two points on a graph with dependent and independent variables, we can use the formulas that use change in variables.



$$\boxed{\text{slope } (rate) = \frac{\text{change in dependent variable } (vertical)}{\text{change in independent variable } (horizontal)}} \quad \text{or} \quad \boxed{slope = \frac{\text{vertical change}}{\text{horizontal change}}}$$

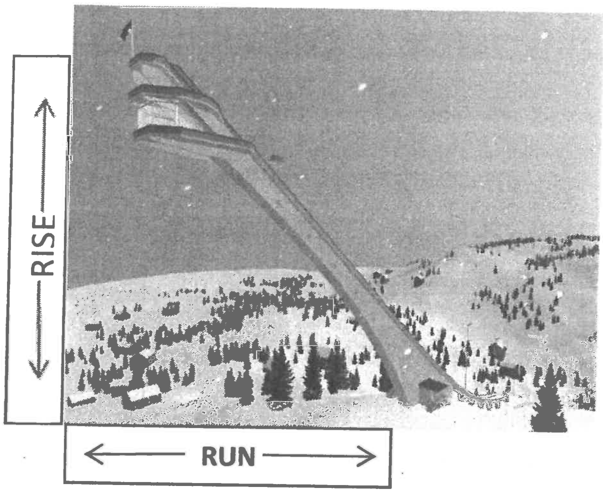
But when we are looking at the slope of tangible objects such as a ramp or a roof, we need to use a different formula. **Slope equals rise over run**. The **rise** is the **vertical height** and the **run** is the **horizontal length**.

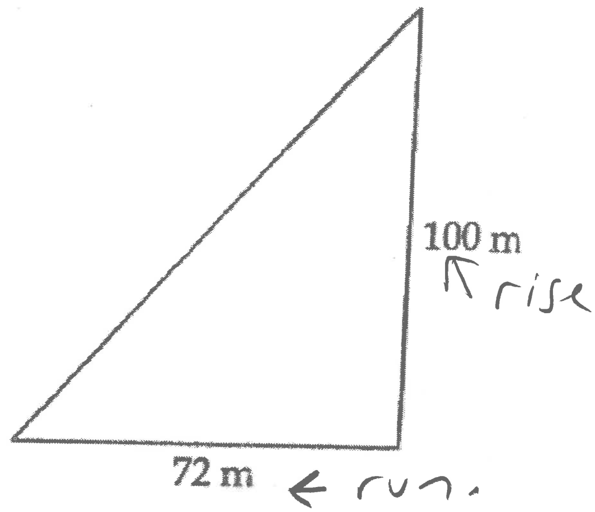
$$\boxed{slope = \frac{rise}{run}}$$

The 3 formulas given for slope are all the same formula, just written in different forms.

You need to use the one easiest for you (and the one that works for the situation.)

EXAMPLE 1: Ski jump hills are quite steep. A picture and a diagram are shown. Find the slope of the ski jump hill. *Finding slope here is the same method as before but we're using measurements instead of data.*





The slope of the hill is 1.4. That means that for every one metre you move horizontally, you move up 1,4m.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{100}{72}$$

$$= 1,388...$$

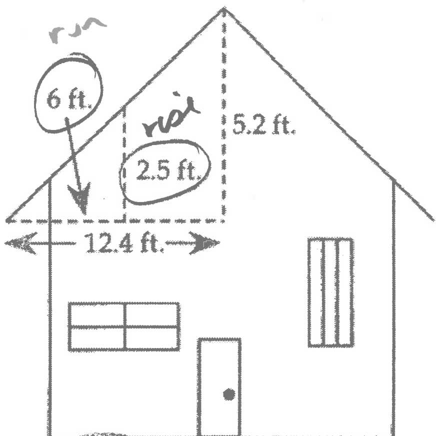
Constant Slope

When using rise over run to calculate the slope of an object, you have to be sure that the whole object has the same slope (a **constant** slope). For example sometimes a roof will change slope partway up - sometimes due to the design, or sometimes just due to a mistake. A constant slope only occurs with a straight line.

Home builders refer to the slope of a roof as its "**pitch**". **To check to see if a roof has a constant slope (or pitch),** divide the roof into intervals (or sections). Determine the slope at each interval. If the slopes are the same, then the roof has a constant slope.

EXAMPLE 2

Fan and Ling are looking for a house and find an old home they like but they are concerned about the amount of work they need to do to repair it. Determine whether the roof has a constant slope, because if it does not they need to rebuild it.



interval 1

rise = 2,5 ft
run = 6 ft

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{2{,}5}{6} = 0{,}4166\ldots \approx 0.42\text{ft}$$

(for every 1 foot horiz, roof rises 0,42 ft.)

interval 2 $\quad \text{slope} = \frac{\text{rise}}{\text{run}}$

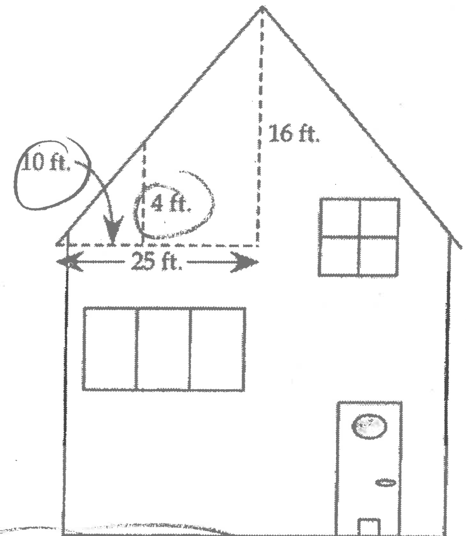
rise = 5,2
run = 12,4

$$= \frac{5{,}2}{12{,}4} = 0{,}4193\ldots \approx 0{,}42\text{ft}$$

Approximately constant slope. (same when rounded even though slightly different when not rounded.) They don't need to rebuild roof.

EXAMPLE 3

Andriko is remodelling a home. He needs to determine whether the roof needs to be repaired. Determine whether or not the roof is straight.



interval 1

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4}{10} = 0.4$$

interval 2

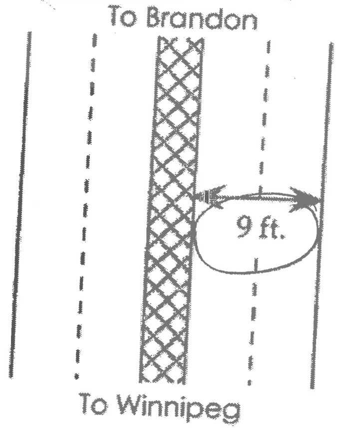
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{16}{25} = 0{,}64$$

Slopes are fairly different.
Roof is not straight.
Andriko will have to repair the roof.

Note: We used multiple points on an object in the same way we used multiple points on the graph of a line to see whether slope is constant.

Slope of Objects Exercise

1. The Trans Canada Highway is slanted (higher in the centre than on the side of the road), so that when it rains, the water runs into the ditches. Heading from Winnipeg to Brandon, the west-bound lanes are 9 feet wide. The outside of the road is 4 inches lower than the inside. What is the road grade (slope) of the highway? Write your answer rounded to two decimal places.



Convert rise + run to same units

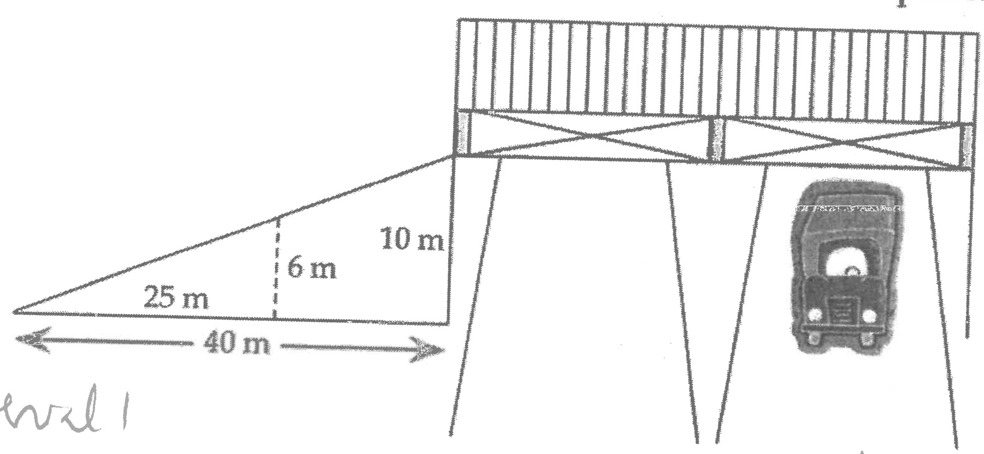
$0.4 \div 12 =$ 0,333.. feet

slope $= \frac{\text{rise}}{\text{run}} = \frac{0,333}{9}$

The road grade is 0.04ft. $= 0,04$

The road rises 0,04 feet for every foot across.

2. A new overpass has been built. Determine whether the incline to the top of it is constant. Write your answer rounded to two decimal places.



interval 1

rise = 6 m run = 25 m

slope $= \frac{\text{rise}}{\text{run}} = \frac{6}{25} = 0.24$

interval 2

rise = 10 m run = 40 m

slope $= \frac{10}{40} = 0.25$

The slopes are quite close but not exactly the same to 2 decimal places. The ramp is not a straight line.

Lesson #6: Rates of Change:

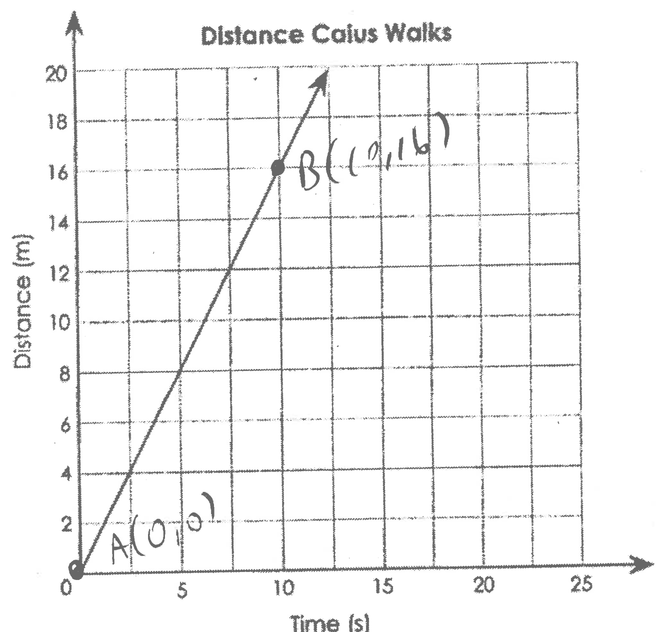
In this lesson, you will

- Describe **slope** as a **rate of change**
- Use **proportional reasoning** to calculate **slope**
- Use **unit analysis** to **interpret data**

Relation to Slope – rate and proportional reasoning

slope = rate of change

EXAMPLE 1: Use the following graph to answer the questions below.



a) Calculate the slope of the graph.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16-0}{10-0} = \frac{16\text{ m}}{10\text{ s}} = 1.6\text{ m/s}$$

or

$$\frac{\text{difference in distance} = 16 - 0 = 16\text{ m}}{\text{difference in time} = 10 - 0 = 10\text{ s}}$$

slope $= \frac{16\text{ m}}{10\text{ s}} = 1.6\text{ m/s}$

b) What does the slope tell us about Caius? The speed Caius is walking is 1.6 m/s.

Note: **speed** is usually **distance** over **time** ex. Km/h, mph, m/s, ft./s The rate of change is the slope of a relation.

c) Without using the graph, calculate how far Caius will walk in **one minute**.

2 methods: a) Use **proportional reasoning using the rate**. Before calculating, be sure you are using the same units in each ration. Remember $\boxed{\text{1 minute = 60 seconds.}}$ *Use the rate of m/s to set up the ratios. Let* **d** *= distance travelled in 60 seconds.*

$$\frac{m}{s} \rightarrow \frac{1.6}{1} = \frac{d}{60}$$

$(1.6)(60) = d(1)$ cross multiply

$96 = d$

Caius will walk 96 m in 1 minute

Include notes about proportional reasoning in your resource sheet.

b) If you have a direction variation, instead of using rate, **use another point from the graph**.

Use point (10, 16).

$$\frac{m}{s} \rightarrow \frac{16}{10} = \frac{d}{60}$$

$16(60) = 10d$ cross multiply

$960 = 10d$ simplify

$96 = d$

Caius will walk 96 m in 1 minute.

Converting Rates of Change – Conversion Ratios

In some cases, the units for slope may not be easily understood because they are not commonly used to describe the rate of change and it is necessary to **convert the rate of change**.

Example 1

Miruna is driving from Brandon to Dauphin. The speed limit on a stretch of highway is 90 km/h. According to her GPS, her driving speed is 30 m/s. Is she speeding?
(*30 m/s is difficult to relate to because we usually describe ~~speed in km/h or~~ mph. Also the speed limit is posted in km/h.)*

Step 1: convert seconds to minutes and then to hours. *There are 60 seconds in a minute and 60 minutes in an hour.*

30 m/sec x 60 = 1800 m/min.
1800 x 60 = 108 000 m/hr.

Step 2: convert metres to kilometres. There are 1000 metres in a kilometre.

108 000 ÷ 1000 = 108 km/hr.

Step 3 re-read the question and answer it in a sentence.

She is speeding.
Her speed is 108 km/hr yet the speed limit is 90 km/hr.

Example 2:

Sergio is negotiating his allowance with his parents for doing work around the house. He would like \$10 per hour. His dad thinks 15¢ per minute is more reasonable, since he doesn't spend very much time doing his chores. Which is the better allowance for Sergio?

Follow the steps in example 1 except this time convert minutes to seconds and then seconds to hours. Once you have converted the 15¢ a minute to the rate per hour, answer the question in a sentence.

15¢ / min × 60

= 900¢ / hr.

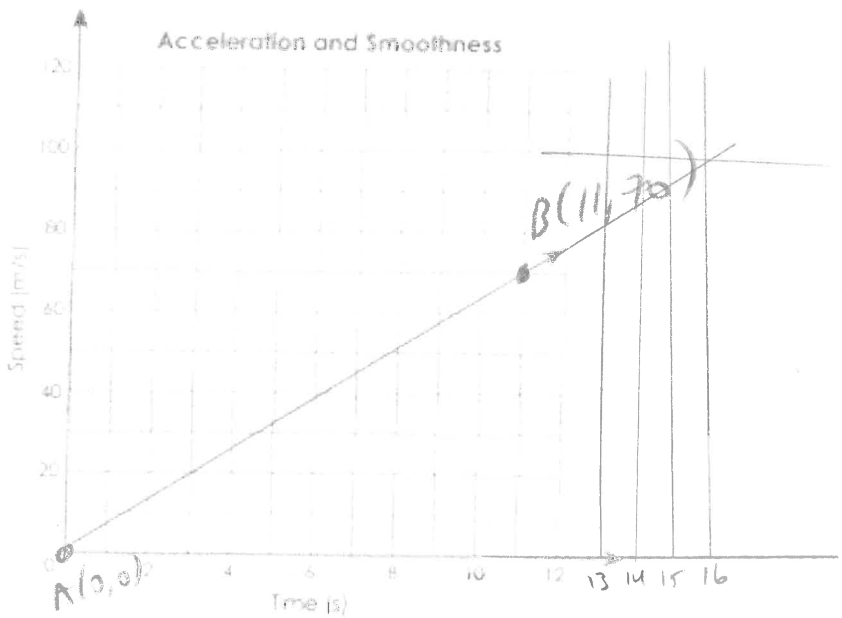
900¢ / hr ÷ 100

= 9\$ / hr.

The better allowance for Sergio is \$10 per hour because 15¢/min equals \$9/hr, which is less.

Exercise: Rates of Change

1. As part of the testing of a new car, a company monitors the acceleration of the car to be sure that it is smooth. The graph below includes their data. Use it to answer the following questions.



a) Calculate the slope of the graph.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{70 - 0}{11 - 0} = \frac{70}{11} = 6.4 \text{ km/hr PER SECOND}$$

We're calculating speed (km/hr) of travel per second

b) What does the slope mean for this situation?

acceleration of the car

Acceleration is 6.4 km/hr per second.

Speed increases by 6.4 km/hr each second.

c) How long would it take the car to reach 100 km/hr? Use two methods. You can extrapolate (extend) the graph to find an approximate answer. You can set up ratios (see p. 48) to find the answer.

(1) extrapolate (extend graph) → about 16 seconds.

(2) We know it is direct variation (passes through (0,0)).

$\frac{km/hr}{sec}$ $\quad \frac{6.4}{1} = \frac{100}{x}$ set up 2 ratios

$\frac{6.4x}{6.4} = \frac{100}{6.4}$ cross multiply divide both sides by 6.4

$x = 15.6$ seconds Simplify.

It would take 15.6 seconds (around 16) to reach 100 km/hr.

2. Telmar is riding his bike at 8 m/s. The speed limit is 30 km/hr. Is he speeding?

8 m/s x 60 = 480 m/min.
48 m/min x 60 = 28 800 m/hr.
28000 ÷ 1000 = 28.8 km/hr
He is not speeding.

3. Canadian highway speed limits are often 100 km/hr. In the US, the highway speed limit is approximately 60 mph. Which is faster? Convert miles to kilometres or kilometres to miles. (1 mi = 1.61 km)

100 km/hr ÷ 1.61 = 62.1 m/hr (more than 60 mph.)
or
60 mi/hr x 1.61 = 96.9 km/hr. (less than 100 km/hr.)
100 km/hr is a little faster.

4. Idra is filling a 4 L bucket with water. The water pours at 15 mL per second. (1 litre = 1000 mL)

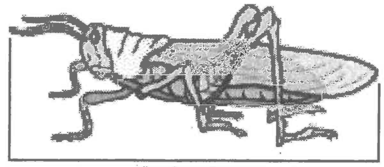
a) Calculate the flow rate in L/min. b) How long will it take to fill the bucket?

15 ml/sec ÷ 1000 = 0.015 l/sec
0.015 l/sec x 60 = 0.9 l/min.
4l ÷ 0.9 l/min = 4.4 min to fill up the bucket.

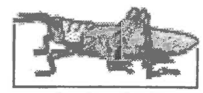
Lesson 7: Scale

Scale Drawings

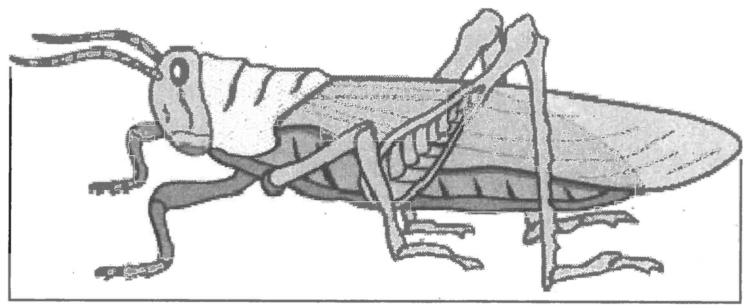
- We use a scale drawing to represent something to big or too small to be drawn at its actual size. A **scale drawing** is **proportional to a life size drawing** of the same object. It is a drawing that shows a **real object** with **accurate sizes** reduced or enlarged by a certain amount (called the **scale factor**). An object may be many times as large as the scale drawing or many times smaller.



Scale of 1:1



Scale of 1:2



Scale of 2:1

- A **scale factor** is a **ratio** of the length of the drawing to the actual length of the object.
- Scale drawing problems are solved using **proportional reasoning** and **finding equivalent ratios**.
- Scale drawings have many applications in everyday life. Examples include: drawing of a floor plan of a house, a blueprint, a map, a photograph, an enlarged diagram of a microscopic image, a model.

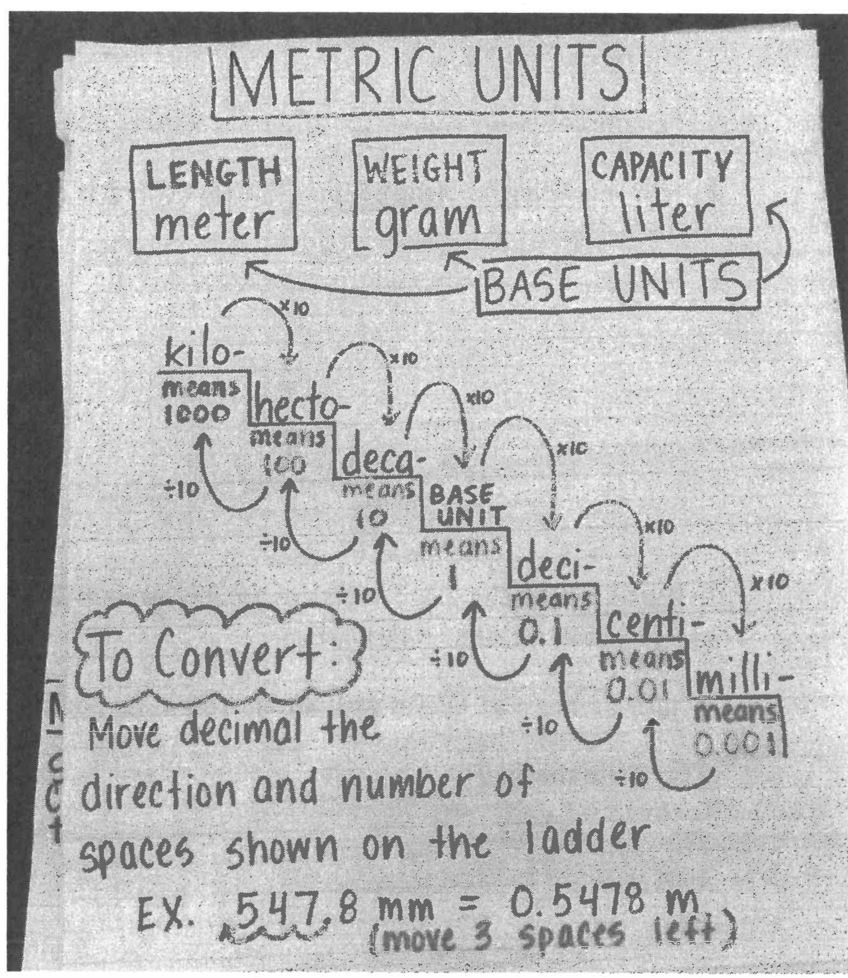
Scale Measurement

Scale drawings involve **measurements.**

Recall the following relationship between units of the metric system.

Starting with millimetres (mm), the metric builds by **multiples of 10.** (eg. 1 cm equals 10 mm.)

When changing to a smaller unit (to the right) you multiply by 10. To change to a larger unit (to the left) you divide by 10.



Example 1: Find the following measures.

(a) 2km = 2000 m

To change from km to m you:

× 1000

move 3 spots to right

× 1 with 3 zeros

$\times 10^3$

(b) 4 cm = 0.04 m

To change from cm to m you:

÷ 100

move 2 spots 2 left

÷ 1 with 2 zeros

$\div 10^2$

(c) 3600 km = ______ cm

360 000 000 cm

To change from km to cm you:

multiply by 100 000

move 5 spots to the right.

$\times 10^5$

×1 with 5 zeros.

(d) 18½ mm= 0.0185 m

18.5

To change from mm to m you:

÷ 1000

move 3 spots to left

$\div 10^3$

Scale Factors

Scale factor is a Ratio:

First number (top number) is measurement on **drawing**

Second number (bottom number) is measurement of actual **real object**

Drawing : object or $\frac{drawing}{real}$

Scale factors can be represented in various ways:

Words:

1 cm represents 50 cm

Ratio - fraction

$\frac{1}{50}$

Ratio – colon (:) between length on drawing and length of real thing

1:50 or 1cm : 50 cm

In your resource sheet, write a definition of scale factor and include how to write it.

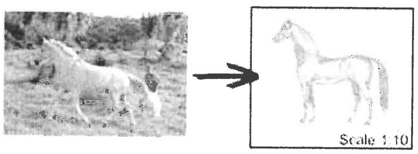
EXAMPLE 2 Write the following scale in two other ways.

a) 1:25 $\frac{1}{125}$ 1cm : 25cm

(*With this form, the unit can be anything.)*

b) 1 cm represents 6 km 1 : 6 000 000 1cm = 6km

(For the ratio as a **fraction**, or as a ratio without units, the **units need to be the same**. *You need to convert 6 km to cm.)* 6km x 10^5 = 60 km x 100 000 = 6 000 000 cm ← 5 zeros



Real Horse 1500 mm high

Drawn Horse 150 mm high

Similar Figures

Similar Figures are figures that have the SAME **SHAPE** but **NOT** the same size. The figures are **proportionally** larger or smaller than each other. A scale drawing is an object that has the same shape as the actual object and is similar to it. This means that all the measures of the diagram and the object are enlarged or reduced by the same ratio. Therefore, given a scale drawing, you can use proportional reasoning to determine the dimensions of the actual objects.

EXAMPLE 3

The following scale drawing represents a dining room table. If the scale is 1:50, determine the actual dimensions of the table, in *metres*. (This means ALL dimensions increase by 50 times.) $\boxed{1m=100cm}$

2.5 cm

5cm

1. First, use your **ruler** to measure the dimensions of the dining room table in the scale drawing.

2. To find the actual **length** of the table, set up a proportion equating two ratios. Each ratio will be in the form of $\boxed{\frac{drawing}{actual}}$.

The **first ratio** is the given scale. The **second ratio** is the **length**.

	scale	length
drawing	1	5
actual	50	x

Let "x" represent the unknown length *in cm* and then in *m*.

$\frac{1}{50} = \frac{5}{x}$

$1(x) = 5(50)$

$x = 250$ cm = length

250cm ÷ 100

length = 2.5 m

3. Cross-multiply to solve for x.

4. Follow the same method as in steps 2 and 3 to find the **width**.

$\frac{1}{50} = \frac{2.5}{x}$

$x = (2.5)(50)$

$x = 125$ cm

125 ÷ 100 = 1.25m

Actual length: 2.5m

Actual width: 1.25m

1.Try it:

The length of an auditorium is **22 m,** and the width is **14 m.** Use a pencil and ruler to draw a scale diagram of the auditorium using the scale factor: **1 cm represents 4 m.**

(Use the steps from p. 53 to set up a proportion to find the length and width of your reduction diagram. Then use a ruler to draw the scale diagram. You need to include the scale factor so that anyone looking at the drawing is able to calculate the actual dimensions of the object.

Length $\frac{\text{drawing}}{\text{actual}}$ $\frac{cm}{m}$ $\frac{1}{4} = \frac{x}{22}$

$22 = 4x$ - Cross multiply

$\frac{22}{4} = \frac{4x}{4}$ - divide by 4 on both sides

$5.5 = x$

Length 5.5 cm

Width $\frac{cm}{m}$ $\frac{1}{4} = \frac{x}{14}$

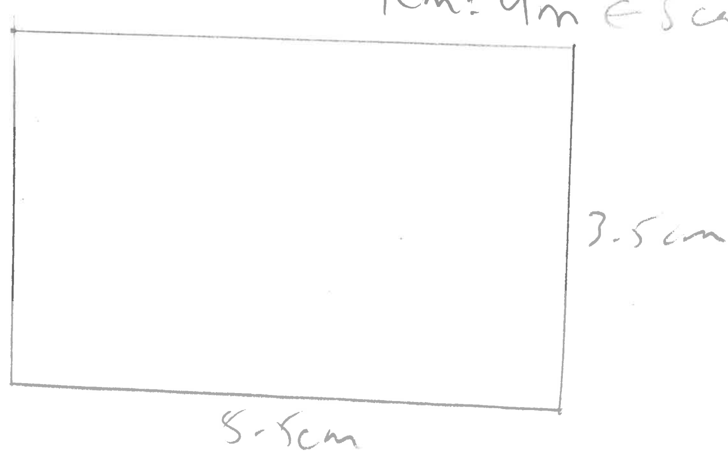
$14 = 4x$

$\frac{14}{4} = \frac{4x}{4}$

$3.5 = x$

Width 3.5 cm.

1cm: 4m ← scale



Exercise: Scale

1. Complete the following chart.

Length in Drawing (cm)	Actual Length (cm)	Scale
6.8 x100	680	1:100
5.2 x 2000	10 400	1: 2000
4.5 x10	45	1:10
9	÷20 180	1:20

2, Represent the following scale factors in two other ways.

a) 1 cm represents 8 m $\frac{1}{800}$ 1:800

b) $\frac{1}{10}$ 1 unit represents 10 units 1:10

c) 1:3 $\frac{1}{3}$ 1 unit represents 3 units

3. A building is 130 m tall. If the height of the building in the scale drawing is 3.25 ~~m~~ cm, what scale factor is used in the drawing?

1 convert building m to cm.
130 m x 10 x 10 = 13000 cm.

$\frac{\text{drawing}}{\text{actual}}$ $\frac{3.25}{13\,000} = \frac{1}{x}$

$\frac{3.25x}{3.25} = \frac{13\,000}{3.25}$

$x = 4000$

scale 1:4000
or
4000 cm ÷ 100 = 40
1cm : 40m.

4. A volleyball court measures 5 m by 8 m. If it is drawn with a scale factor of 1:300, what are the dimsenions of the volleyball court in the scale drawing?

1 Convert m to cm. 5m x 100 = 500cm, 8m x 100 = 800cm.

length $\frac{\text{drawing}}{\text{actual}}$ $\frac{1}{300} = \frac{x}{800}$

$\frac{300x}{300} = \frac{800}{300}$

$x = 2.7$ cm.

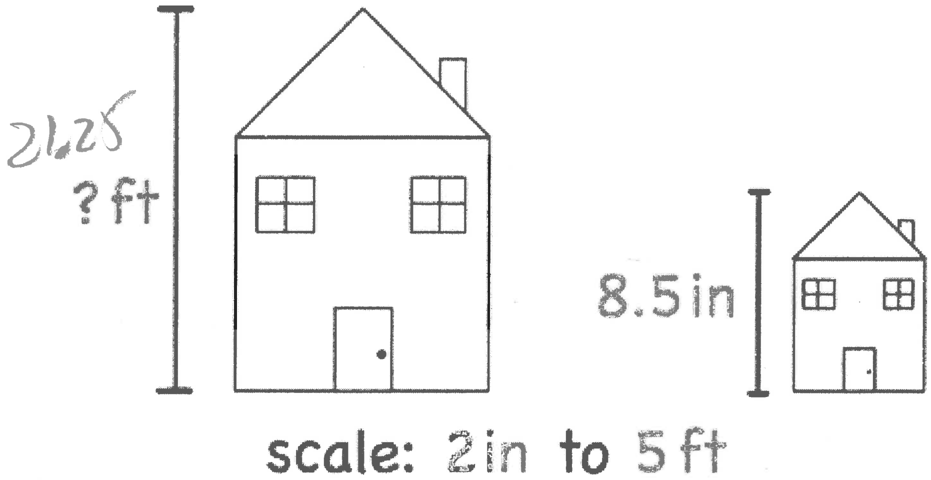
width $\frac{1}{300} = \frac{x}{500}$

$\frac{300x}{300} = \frac{500}{300}$

$x = 1.7$ cm

Scale drawing 2.7cm x 1.7cm.

5. Calculate the actual height of the house.



$$\frac{\text{drawing (in)}}{\text{actual (ft)}} \quad \frac{2}{5} = \frac{8.5}{x}$$

$$\frac{2x}{2} = \frac{42.5}{2}$$

$x = 21.25$ ft. - height of house

6. A contractor has a blueprint for a house drawn to the scale 1 in: 3 ft. One wall of the house will be 12 feet long when it is built. How long is the wall on the blueprint?

$$\frac{\text{drawing}}{\text{actual}} \quad \frac{\text{in}}{\text{ft}} \quad \frac{1}{3} = \frac{x}{12}$$

$$\frac{3x}{3} = \frac{12}{3}$$

$x = 4$ in. ← wall in blueprint